Problems Day 55, T 4/30/2024
Topic 27: Linear phase portraits (day 2+)
Jeremy Orloff

Problem 1. Let $A = \begin{bmatrix} 3 & 4 \\ -4 & 3 \end{bmatrix}$. Sketch a phase portrait of the system $\mathbf{x}' = A\mathbf{x}$.

Name the type of critical point at the origin and give its dynamical stability.

Problem 2. A system $\mathbf{x}' = A\mathbf{x}$ has general solution $\begin{bmatrix} x \\ y \end{bmatrix} = c_1 \begin{bmatrix} \cos t \\ -\sin t \end{bmatrix} + c_2 \begin{bmatrix} \sin t \\ \cos t \end{bmatrix}$

Sketch a phase portrait of the system. Name the type of critical point at the origin and give its dynamical stability.

Problem 3. For this problem, we have 2×2 system $\mathbf{x}' = A\mathbf{x}$. For each one, give the type of critical point at $(0,0)$, its dynamical stability and the structural stability of the system.

(a) $\operatorname{tr} A = -3, \quad \det A = 2$

(b) $\operatorname{tr} A = -3, \quad \det A = -2$

(c) $\operatorname{tr} A = -2, \quad \det A = 1$

Problem 4. A system $\mathbf{x}' = A\mathbf{x}$ has solution $\mathbf{x}(t) = c_1 e^{2t} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2 e^{2t} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = e^{2t} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$.

Sketch a phase portrait.

Problem 5. A system $\mathbf{x}' = A\mathbf{x}$ has solution $\mathbf{x}(t) = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 e^{-2t} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

Sketch a phase portrait.

Problem 6. A system $\mathbf{x}' = A\mathbf{x}$ has solution $\mathbf{x}(t) = c_1 e^{2t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 e^{2t} \left(t \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right)$.

Sketch a phase portrait.

MIT OpenCourseWare

https://ocw.mit.edu

ES.1803 Differential Equations

Spring 2024