

Problems Day 57, R 5/2/2024

Topic 28: Nonlinear Systems

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Problem 1. Consider the system

$$\begin{aligned}x' &= 3x - x^2 - xy \\y' &= y - y^2 + xy\end{aligned}$$

- (a) Find the critical points
- (b) Linearize at each critical point. Say what this implies about the nonlinear system.
- (c) On the xy -plane, show each of the critical points and the linear approximation near it.
- (d) Tie Part (c) together into a phase-portrait of the nonlinear system.
- (e) Suppose x, y represent populations of two species, tell a story about their interaction.

Problem 2. Consider the system

$$\begin{aligned}x' &= 2x - 3xy \\y' &= -y + 2xy\end{aligned}$$

Answer the same questions as in Problem 1.

Note: One of the critical points is not structurally stable. So you will have to entertain several possibilities for its type.

(e) As in Problem 1, this is a predator-prey relationship, with $x = \text{prey}$ and $y = \text{predator}$. Unlike Problem 1, in the absence of y , x grows exponentially. And, in the absence of x , y decays exponentially.

Because the critical point at $(1/2, 2/3)$ could be a center, spiral source or spiral sink, we can't say exactly how the populations evolve.

Note: With more work (which we won't do in ES.1803), it is possible to show that $(1/2, 2/3)$ is a nonlinear center. So, the populations will circle around the critical point.

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ES.1803 Differential Equations

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