

## Problems Day 6, M 2/12/2024

Topic 4: Complex numbers (day 1)

Jeremy Orloff

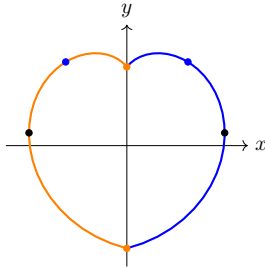
**Problem 1.** Let  $z_1 = 2 + 5i$ ,  $z_2 = 1 + 3i$

- (a) Compute  $z_1 + z_2$ ,  $z_1 \cdot z_2$ ,  $z_1 \cdot \bar{z}_1$ ,  $|z_1|$ ,  $\text{Arg}(z_1)$ .
- (b) Find  $\text{Re}(z_1)$ ,  $\text{Im}(z_1)$ .
- (c) Let  $z = x + iy$ . Compute  $z \cdot \bar{z}$ .

**Problem 2.** Find  $r$  and  $\theta = \text{Arg}(z)$  for  $z = i$ ,  $z = 1 - i$ ,  $z = 5(1 - i)$ .

**Problem 3.**

- (a) Write  $\frac{i}{2}$  in polar form.
- (b) Consider the diagram



Multiply the diagram by  $i/2$ , i.e., sketch the resulting image.

**Problem 4.** Show  $\overline{e^{i\theta}} = e^{-i\theta}$ .

**Problem 5.** Compute  $(1 + \sqrt{3}i)^{10}$ . (Use polar form.)

**Problem 6.** Compute  $I = \int e^x \cos(5x) dx$ .

**Problem 7.**

- (a) Find the fifth roots of 1. Draw a picture.
- (b) Find the fifth roots of  $1 + i$ . Draw a picture.

**Problem 8.**

- (a) Draw the trajectory of  $z = e^{it}$
- (b) Draw the trajectory of  $z = te^{it}$
- (c) Plot the points  $e^{ij\pi/4}$ , for  $j = 0, 1, 2, 3 \dots$

**Problem 9.**

- (a) Write  $\sin t$  and  $\cos t$  in terms of  $e^{it}$  and  $e^{-it}$ .

(b) Find all real-valued functions of the form  $f(t) = c_1 e^{it} + c_2 e^{-it}$ , where  $c_1, c_2$  are complex constants.

**Problem 10.** Find all the roots of  $x^4 + x^2 = 0$ .

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ES.1803 Differential Equations

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