ES.1803 Practice Questions – Quiz 1, Spring 2024
Covers Topics 1-3

This will probably take longer than 1 hour; the actual quiz will be considerably shorter. Not every possible question is covered. For more questions, you can also look at the in-class problems, problem section worksheets and psets.

Be wise in carrying out computations. If you get a problem to a point where you are sure –*emphasize sure*– you can carry out the computation, then stop and go on to the next problem. At the end you can check the solutions to see that you had correctly set up the computation.

Problem 1.
Solve the DE $\dfrac{dy}{dx} = xy^2$.

Problem 2.
Solve the following DE. $\cos(x)\dfrac{dy}{dx} + \sin(x)\cos(y) = 0$

Problem 3.
A species S in a constrained envrironment cannot grow exponentially forever. Call the population over time $x(t)$. Assume the growth rate is not constant, but depends on x. Specifically,

$$\text{growth rate} = aM - ax.$$

(a) What are the units on M and a

(b) Model the population with a DE.

Problem 4.
A salt solution of strength 2 grams per liter is flowing into a 50 liter tank, and the solution in the tank is being pumped out, both at the (same) time-varying rate of $\dfrac{1}{1+t}$ liters per minute.

(a) With the usual instantaneous-mixing assumptions, derive the DE for the rate of change of the amount of salt $x = x(t)$ (in grams) in the tank with respect to time (in minutes).

(b) Solve this DE explicitly for $x = x(t)$ with the IC $x(0) = 0$ (i.e., starting off with pure water in the tank).

(c) Using both (i) the solution $x = x(t)$ found in Part (b) and (ii) an argument from 'first principles' (i.e., directly from the physical situation described here), answer the following question: What happens to the amount of salt in the tank in the long-run over time? In particular, does it approach a final limiting value? If so, what is this value?

(The idea is to do both (i) and (ii) and show they give the same prediction for the long-term behavior.)

Problem 5.
Assume y_1 is a solution to the DE $y' + e^t y = 0$. Also assume that y_p is a solution to the

DE $y' + e^t y = t^4$. Prove that all the functions $y = y_p + cy_1$, where c is any constant, are also solutions to $y' + e^t y = t^4$.

Problem 6.

Suppose that the functions $y_1 = t$ and $y_2 = \frac{1}{t}$ both satisfy a certain inhomogeneous first-order linear DE. Write down the general solution to the DE.

End of practice quiz

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