

ES.1803 Practice Questions – Quiz 4, Spring 2024 Covers Topics 13-16

This will probably take longer than 1 hour; the actual quiz will be considerably shorter. Not every possible question is covered. For more questions, you can also look at the in-class problems, problem section worksheets and psets.

Be wise in carrying out computations. If you get a problem to a point where you are sure *–emphasize sure–* you can carry out the computation, then stop and go on to the next problem. At the end you can check the solutions to see that you had correctly set up the computation.

Problem 1.

Define all the following sets.

(i) $S_1 = \{C_1 e^{-t} + C_2 \cos(5t)\}$

(ii) $S_2 = \{C_1(e^{-t} + e^{-7t}) + C_2(e^{-t} - e^{-7t})\}$

(iii) $S_3 = \{e^{-t}, e^{-7t}\}$

(iv) Let A and B both be $m \times n$ matrices. Let

$$S_4 = \{\text{All pairs of vectors } (\mathbf{x}, \mathbf{y}) \mid A\mathbf{x} = B\mathbf{y}\}$$

(v) $S_5 =$ the span of the rows of a matrix A .

(vi) $S_6 = \left\{ \begin{bmatrix} 3 \\ x + y \end{bmatrix} \right\}$

(vii) $S_7 = \left\{ \begin{bmatrix} x - 3 \\ x + 3 \end{bmatrix} \right\}$

(viii) $S_8 = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \mid x \geq 0, y \geq 0 \right\}$

(a) Which of the sets are closed under addition.

(b) Which of the sets are closed under scalar multiplication.

(c) Which of the sets are vector spaces.

Problem 2.

Let $A = \begin{bmatrix} 1 & 2 & 3 & 6 \\ 1 & 2 & 1 & 4 \\ 0 & 0 & 1 & 1 \end{bmatrix}$.

(a) Find the reduced echelon form of A .

(b) What is the rank of A ?

(c) Find a basis for the null space of A .

(d) Find a basis for the column space of A .

(e) Find a matrix with the same reduced echelon form but such that $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ are in its column space.

Problem 3.

$$\text{Let } A = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

- (a) Find a basis for $\text{Null}(A)$
- (b) Is A invertible?
- (c) Use Part (a) to give one eigenvalue and corresponding eigenvector of A .
- (d) Find the other two eigenvalues of A .

Problem 4.

- (a) Consider the matrix $A = \begin{bmatrix} 6 & 5 \\ 1 & 2 \end{bmatrix}$. For which values of a is the matrix $A - aI$ singular? (Singular means it doesn't have an inverse.)
- (b) For each value of a in Part (a): find the null space of $A - aI$.

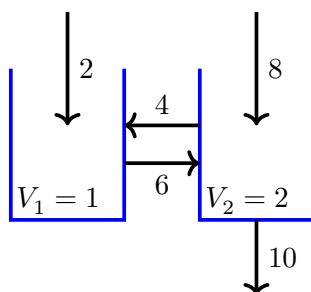
Problem 5.

Consider the system $x' = 5x - 6z$, $y' = 2x - y - 2z$, $z' = 4x - 2y - 4z$.

Rewrite this system of DEs in matrix form $\mathbf{x}' = A\mathbf{x}$.

Problem 6.

Suppose we have a two compartment system with flow rates and volumes (in some compatible units) as shown. Suppose the concentration of solute in the inflows are a and b for Tanks 1 and 2 respectively.



- (a) Give the system of DEs modeling the amounts of solute $x(t)$, $y(t)$ in Tanks 1 and 2. Write your answer in matrix form.
- (b) Find a particular solution \mathbf{x}_p to this inhomogeneous DE by guessing a **constant** solution. (Give your answer in terms of a and b .)

End of practice quiz

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ES.1803 Differential Equations

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