ES.1803 Practice Questions – Quiz 4b, Spring 2024
Covers Topics 13-14

This will probably take longer than 1 hour; the actual quiz will be considerably shorter. Not every possible question is covered. For more questions, you can also look at the in-class problems, problem section worksheets and psets.

Be wise in carrying out computations. If you get a problem to a point where you are sure –*emphasize sure*– you can carry out the computation, then stop and go on to the next problem. At the end you can check the solutions to see that you had correctly set up the computation.

Problem 1.
Solve this system of linear equations. How many methods can you think of to solve this system?

$$\begin{aligned} x + y &= 5 \\ 3x + 2y &= 7 \end{aligned}$$

Problem 2. Let $R = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ and suppose R is the reduced row echelon form for A.

(a) What is the rank of A?

(b) Find a basis for the null space of A.

(c) Suppose the column space of A has basis $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$. Find a possible matrix for A. That is, give a matrix with RREF R and the given column space.

(d) Find a matrix with the same reduced echelon form but such that $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ are in its column space.

Problem 3. Consider the following system of equations:

$$\begin{aligned} x + \ \ y + \ \ z &= 5 \\ x + 2y + 3z &= 7 \\ x + 3y + 6z &= 11 \end{aligned}$$

(a) Write this system of equations as a matrix equation.

(b) Use row reduction to get to row echelon form. What is the solution set?

Problem 4.
(a) Try to solve the following equation using row reduction:

$$\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

At the end of the row reduction process, was the augmented column pivotal or free? Is this related to the absence of solutions?

(b) Find a new vector $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ such that $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ has a solution. What is the set of all solutions of this new equation?

Problem 5. Let $A = \begin{bmatrix} 1 & 2 & 3 & 1 & 2 \\ 2 & 4 & 6 & 2 & 4 \\ 0 & 0 & 10 & 3 & 6 \end{bmatrix}$. Put A in row reduced echelon form. Find the rank, a basis of the column space, a basis of the null space, and the dimension of each of the spaces.

Problem 6. (a) Suppose we have a matrix equation

$$\begin{bmatrix} \bullet \\ \bullet \end{bmatrix} \begin{bmatrix} \bullet & \bullet \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & x \end{bmatrix}$$

Can you specify x?

(b) Suppose we have a matrix equation

$$\begin{bmatrix} \bullet & 3 \\ 4 & \bullet \\ \bullet & 6 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Can you specify the $\bullet$'s?

Problem 7. Suppose we have a matrix equation $A = \begin{bmatrix} 1 & x & 2 \\ 3 & y & 4 \\ 5 & z & 6 \end{bmatrix}$. All we know about A is that $\mathrm{Null}(A)$ is nontrivial. What can we say about $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$?

Problem 8. For what values of y, z are the columns of the matrix $\begin{bmatrix} 1 & 1 & 2 \\ 3 & 4 & y \\ 5 & 6 & z \end{bmatrix}$ linearly independent?

Problem 9. Let $A = \begin{bmatrix} 1 & 4 & 7 \\ 3 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$:

(a) Find the row reduced echelon form of A; call it R.

(b) The last column of R should be a linear combination of the first columns in an obvious way. Find a vector $\mathbf{x}$, such that $R\mathbf{x} = \mathbf{0}$, which expresses this linear relationship.

(c) Verify that the same relationship holds among the columns of A.

Problem 10. This continues the previous problem. Now suppose we want to solve $A\mathbf{x} = \mathbf{b}$.

(a) For what $\mathbf{b}$ is this possible?

(b) For those $\mathbf{b}$ found in Part (a), describe the general solution to $A\mathbf{x} = \mathbf{b}$.

Problem 11. Suppose that the reduced echelon form of the 4×6 matrix B is

$$R = \begin{bmatrix} 0 & 1 & 2 & 3 & 0 & 5 \\ 0 & 0 & 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) Find a basis for $\text{Null}(B)$.

(b) Suppose we write $B = \begin{bmatrix} \mathbf{b_1} & \mathbf{b_2} & \mathbf{b_3} & \mathbf{b_4} & \mathbf{b_5} & \mathbf{b_6} \end{bmatrix}$, where $\mathbf{b_1}$ etc. are the columns? What can you say about these columns.

End of practice quiz