ES.1803 Problem Section 14, Spring 2024

Topic 31 (nonlinear mechanical systems) is not officially part of the course, but these problems are fun and will give you more practice with nonlinear systems.

Problem 31.1. Nonlinear Spring
The following DE models a nonlinear spring:

$$m\ddot{x} = -kx + cx^3 \qquad \begin{cases} \textbf{hard if } c < 0 & \text{(cubic term adds to linear force)} \\ \textbf{soft if } c > 0 & \text{(cubic term opposes linear force).} \end{cases}$$

(a) Convert this to a companion system of first-order equations.

(b) Sketch a phase portrait of the system for both the hard and soft springs. You can use the fact that the linearized centers are also nonlinear centers. (This follows from energy considerations.)

(c) (Challenge! For anyone who is interested. This is not part of the ES.1803 syllabus.) Find equations for the trajectories of the system.

Problem 31.2. The **damped nonlinear spring** has equation

$$m\ddot{x} = -kx + cx^3 - b\dot{x}.$$

(a) Convert it to a system of first-order equations.

(b) Sketch a phase portrait for both the hard and soft springs.

ES.1803 Differential Equations

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