ES.1803 Problem Section 3, Spring 2024

Problem 5.1. **(a)** Solve $x'' - 8x' + 7x = 0$ using the characteristic equation method.

(b) Solve $x'' + 2x' + 5x = 0$ using the characteristic equation method.

(c) Assume the polynomial $r^5 + a_4r^4 + a_3r^3 + a_2r^2 + a_1r + a_0 = 0$ has roots

$$0.5, \quad 1, \quad 1, \quad 2 \pm 3i.$$

Give the general real-valued solution to the homogeneous constant coefficient DE

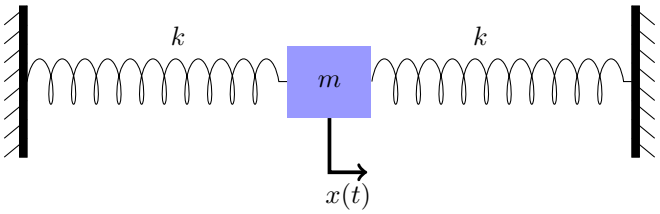
$$x^{(5)} + a_4x^{(4)} + a_3x^{(3)} + a_2x'' + a_1x' + a_0x = 0.$$

Problem 5.2. **(Unforced second-order physical systems)**
The DE $x'' + bx' + 4x = 0$ models a damped harmonic oscillator. For each of the values $b = 0, 1, 4, 5$ say whether the system is undamped, underdamped, critically damped or overdamped.

Sketch a graph of the response of each system with initial condition $x(0) = 1$ and $x'(0) = 0$. (It is not necessary to find exact solutions to do the sketch.)

Say whether each system is oscillatory or non-oscillatory.

Problem 5.3. In the spring system below, both springs are unstretched when the position of the mass is $x = 0$, which is exactly in the middle. Write down a DE modeling the position of the mass over time.



Problem 5.4. State and verify the superposition principle for $mx'' + bx' + kx = 0$, (m, b, k constants).

Problem 5.5. A constant coefficient, linear, homogeneous DE has characteristic roots

$$-1 \pm 2i, \; -2, \; -2, -3 \pm 4i.$$

(a) What is the order of the DE? (Notice the $\pm$ in the list of roots.)

(b) What is the general, real-valued solution.

(c) Draw the pole diagram for this system. Explain why it shows that all solutions decay exponentially to 0. What is the exponential decay rate of the general solution?

ES.1803 Differential Equations

Spring 2024