ES.1803 Problem Section 4, Spring 2024

Problem 6.1. Let $P(D) = D^2 + 6D + 5I$. Find the general real-valued solution to each of the following.

(a) $P(D)x = e^{-2t}$.

(b) $P(D)x = \cos(3t)$.

(c) $P(D)x = e^{2t}\cos(3t)$.

(d) $P(D)x = e^{-t}$.

Problem 6.2. **(a)** Solve $x'' + 4x = \cos(\omega t)$ for all possible values of ω.

(b) Plot the graph of your particular solution for $\omega = 2$.

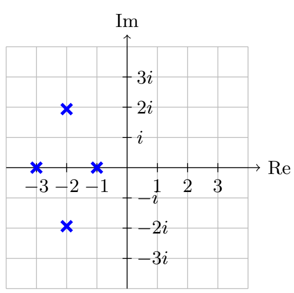
Problem 7.3. Solve $x' + 3x = t^2 + t$.

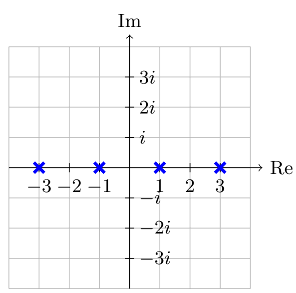
Problem 8.4. Is the system $x'' + x' + 4x = 0$ stable?

Problem 8.5. Is a 4th order system with roots $\pm 1, -2 \pm 3i$ stable. Which solutions to the homogeneous DE go to 0 as $t \to \infty$?

Problem 8.6. For what k is the system $x' + kx = 0$ stable?

Problem 8.7. **(a)** The pole diagram below on the left shows the characteristic roots of the system $P(D)x = 0$.





Left: pole diagram for Part (a). Right: diagram for Part (b)

(i) What is the order of the system?

(ii) Is the system stable?

(iii) Is the system oscillatory?

(iv) What is the exponential decay rate for the general solution?

(b) Repeat Part (a) for the pole diagram on the right.

Problem 8.8. Consider the following systems.

(i) $x'' + x' + 4x = 0$
(ii) A fourth-order system with roots ± 1, $-2 \pm 3i$
(iii) $x' + 3x = 0$.

Draw the pole diagram for each of these systems and say how it relates to the stability of the system.

Extra problems if time.

Problem 6.9. (Sinusoidal response formula (SRF))
Let $P(D) = D^2 + 4D + 3$. Find a solution to $P(D)x = \cos(2t)$

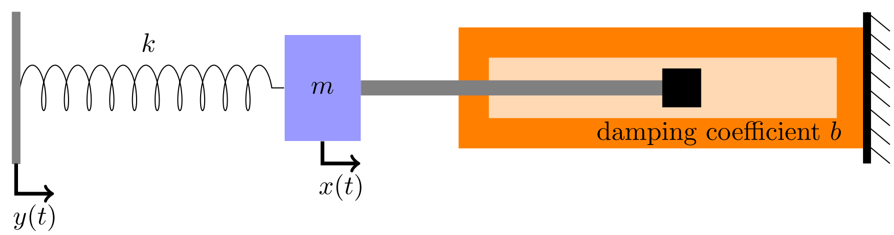
Problem 6.10. Solve $P(D)x = x'' + 4x' + 5x = e^{-t}\cos 2t$.

Do this using complex replacement. Give the general solution.

Problem 6.11. (a) Show directly from the definition that $P(D) = D^3 + 6D^2 + 7I$ is a linear operator.

(b) Say to yourself: "Checking linearity is always easy. You just have to remember to ask."

Problem 6.12. Driving through the spring. Suppose the spring-mass-dashpot is driven by a mechanism that positions the end of the spring at $y(t)$ as shown. As before, $x(t)$ is the position of the mass. We calibrate x and y so that $x = 0$, $y = 0$ is an equilibrium position of the system.



Give the DE modeling the position $x(t)$ of the mass. Assume, m, k, b, x, y are in compatible units.

Problem 6.13. Let $P(D) = D^2 + 4D + 6I$. Solve $P(D)x = \cos(2t)$.

Problem 7.14. Find a particular solution to $x'' + x' = t^4$.

Write down the system of equations for A, B, C, D, E, but don't bother solving.

Problem 7.15. (Example from Topic 7 notes.)
Solve $y'' + 5y' + 4y = 2t + 3$ by the method of undetermined coefficients.