# ES.1803 Problem Section 9, Spring 2024

**Problem 20.1.** Compute the following integrals.

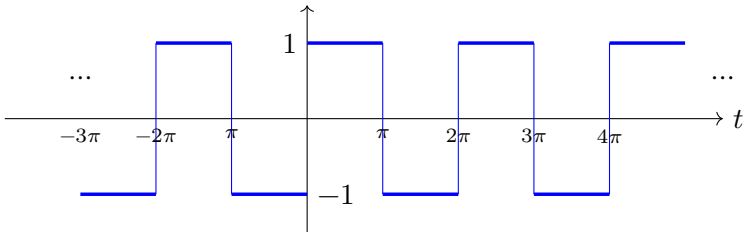
**(a)** $\displaystyle\int_{-\infty}^{\infty} \delta(t) + 3\delta(t-2)\,dt$

**(b)** $\displaystyle\int_{1}^{5} \delta(t) + 3\delta(t-2) + 4\delta(t-6)\,dt.$

**Problem 20.2.** Solve $x' + 2x = \delta(t) + \delta(t-3)$ with rest IC

**Problem 20.3.** (Second-order systems) Solve $4x'' + x = 5\delta(t)$ with rest IC.

**Problem 20.4. Derivative of a square wave**
The graph below is of a function $\mathrm{sq}(t)$ (called a square wave). Compute and graph its generalized derivative.




Graph of $\mathrm{sq}(t)$ = square wave

**Problem 20.5.** Compute the following integrals.

**(a)** $\displaystyle\int_{0^-}^{\infty} \cos(t)\delta(t) + \sin(t)\delta(t-\pi) + \cos(t)\delta(t-2\pi)\,dt.$

**(b)** $\displaystyle\int \delta(t)\,dt.$ (Indefinite integral)

**(c)** $\displaystyle\int \delta(t) - \delta(t-3)\,dt.$ Graph the solution

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**Extra problems if time.**

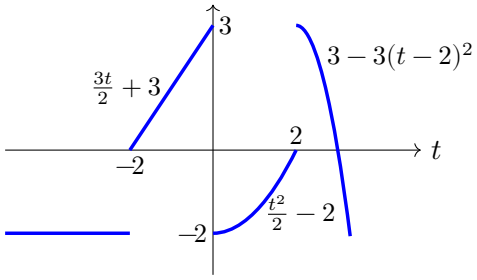
**Problem 20.6.** Solve $x' + 2x = \delta(t)$ with rest IC

**Problem 20.7.** **(a)** Solve $2x'' + 8x' + 6x = \delta(t)$ with rest IC.

**(b)** Plug your solution into the DE and verify that it is correct

**Problem 20.8.** Solve $x' + 3x = \delta(t) + e^{2t}u(t) + 2\delta(t-4)$ with rest IC.
(The $u(t)$ is there to make sure the input is 0 for $t < 0$.)

**Problem 20.9.** The graph of the function $f(t)$ is shown below. Compute the generalized derivative $f'(t)$. Identify the regular and singular parts of the derivative.



ES.1803 Differential Equations

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