

ES.1803 Problem Section Problems for Quiz 1, Spring 2024

Topic 1: Introduction, modeling, separation of variables

Problem 1.1. (Here's the second geometry example in the notes for Topic 1.)

$y = y(x)$ is a curve in the first quadrant. The part of the tangent line in the first quadrant is bisected by the point of tangency. Find and solve the DE for this curve.

Problem 1.2. Consider the family of all lines whose y -intercept is twice the slope.

(a) Find a DE which has this family as its solutions.

(b) Find the orthogonal trajectories to the curves in Part (a). That is, find a family of functions whose graphs intersect all the lines in Part (a) orthogonally.

(c) Sketch both families.

Problem 1.3. You deposit money in a bank at the rate of \$1000/year. The money earns (continuous) 8% interest. Construct a DE to model the amount of money in the bank as a function of time; then solve the DE. Assume that at time 0 there is no money in the bank.

Problem 1.4. Solve $y' = f(x)$, $y(a) = y_0$.

Problem 1.5. Solve $y' = \sin(x^2)$, $y(0) = 1$. Give the solution as a definite integral. (Note, you can't do the integral, but the solution is perfect for numerical computation by computer.)

Problem 1.6. Solve $\frac{du}{dt} = \sin t \cos^2 u$, $u(0) = 0$.

Problem 1.7. (From Topic 1 notes.) Solve $\frac{dy}{dx} = xy$.

Problem 1.8. (From Topic 1 notes.) Solve $\frac{dy}{dx} = x^3y^2$.

Problem 1.9. (From Topic 1 notes.) Solve $y' + p(x)y = 0$.

Topic 2: Linear systems, input-response

Problem 2.10. (Linear homogeneous)

(a) Solve $y' + ky = 0$.

(b) Solve $y' + ty = 0$.

Problem 2.11. (Linear inhomogeneous)

- (a) Solve $y' + 2y = 2$.
- (b) Solve $y' + 2y = 2t$.
- (c) Solve $y' + 2y = 5 + 2t$ using the earlier parts of this problem and superposition.

Problem 2.12. (IVP using definite integrals)

Solve $xy' - e^x y = 0$, $y(1) = 2$ using definite integrals.

Problem 2.13. Solve $y' + 2y = 2$; $y(1) = 1$.

Problem 2.14. Show that $y' + y^2 = q$ does not satisfy the superposition principle.

Topic 3: Input response continued

Problem 3.15. Solve the DE $x' + 2x = f(t)$, $x(0) = 0$, where $f(t) = \begin{cases} 6 & \text{for } 0 \leq t < 1 \\ 0 & \text{for } 1 \leq t < 2 \\ 6 & \text{for } 2 \leq t. \end{cases}$

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