Name _____

ES.1803 Quiz 1, Spring 2024

5 problems, No books, notes or calculators.

Variation of parameters formula: $x(t) = x_h(t) \int \frac{q(t)}{x_h(t)}\, dt + C x_h(t)$.

Problem 1. (15 points)
Find the general solution to $\dfrac{dx}{dt} + tx = t$.

Problem 2. (15 points)
(a) (10) Find the solution to the initial value problem: $\dfrac{dx}{dt} + 3x = 5, \quad x(2) = 7$.

(b) (5) Solve the following differential equation with initial condition. **Hint:** Your answer to Part (a) will help considerably.

$$\frac{dx}{dt} + 3x = \begin{cases} 5 & \text{for } t < 2 \\ 0 & \text{for } 2 < t \end{cases}, \qquad x(2) = 7.$$

Problem 3. (15 points)
(a) (10) A rectangular tank has cross-sectional area 4 m^2. Water flows into the top of the tank at a constant rate b in $\frac{m^3}{\text{min}}$, and out of the bottom at a rate proportional to the height of water in the tank.

Give the DE which governs the height of water in the tank. Be sure to define and give units for any symbols you introduce. (**Hint:** volume = height $\times$ cross-sectional area.)

Do not solve the DE.

(b) (5) Assuming that the tank in Part (a) has sufficient capacity, the height of the water in the tank will reach an equilibrium. What is the equilibrium height?

Problem 4. (15 points)
Consider the family of curves $y = Cx^2$. Find the family of orthogonal trajectories.

Hint: Isolate the C before differentiating.

Problem 5. (10 points)
(a) (5) True or false: Multiplying any solution of $\ddot{y} + 3t\dot{y} + 5y = t$ by 7 produces another solution to the same differential equation.

You need to give a short explanation of your reasoning.

(b) (5) The equation $\ddot{y} + \left(4 - \dfrac{1}{t}\right)\dot{y} + \left(4 - \dfrac{2}{t} - \dfrac{3}{t^2}\right) y = f(t)$ has two solutions:

$$y_1(t) = e^{2t}t^3 + e^{-2t}t^3 \quad \text{and} \quad y_2(t) = e^{2t}t^3 - e^{-2t}t^3$$

Give one nontrivial solution to the homogeneous equation

$$\ddot{y} + \left(4 - \frac{1}{t}\right)\dot{y} + \left(4 - \frac{2}{t} - \frac{3}{t^2}\right) y = 0.$$

ES.1803 Differential Equations

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