Name _____

ES.1803 Quiz 2, Spring 2024

5 problems, No books, notes or calculators.

Problem 1. (50 points)
The differential operator in this problem is: $P(D) = 2D^2 + 2D + 5I$

Hint: The roots of $2r^2 + 2r + 5$ are $-\frac{1}{2} \pm \frac{3}{2}\,i$.

(a) (10) Find the general *real-valued* solution to $P(D)x = e^{-2t}$.

(b) (10) Find the general *real-valued* solution to $P(D)x = 15t - 14$.

(c) (10) Find the periodic solution to $P(D)x = \omega\cos(\omega t)$.

(d) (5) Find the periodic solution to $P(D)x = \sum_{n=1}^{100} n\cos(nt)$.

(e) (5) Find the general real-valued solution to $(D - 3I)(2D^2 + 2D + 5I)x = 0$;

(f) (10) Find a particular solution to $x'' + 9x = \cos(3t)$

Problem 2. (10 points)
Find all the roots of the equation $r^4 = -16$. Give them in the form $a + bi$.

Use this to give the general real-valued solution of $(D^4 + 16I)x = 0$.

If you can't find the roots, then, for 4 points, you can solve the DE pretending the roots are $-2 \pm 3i$, $-4 \pm 5i$.

Problem 3. (10 points)
Consider the equation $x'' + 2x' + kx = 0$.

(a) (5) Give the range of k for which all non-zero solutions are oscillatory.

(b) (5) Give the range of k for which all solutions go to 0 as $t \to \infty$.

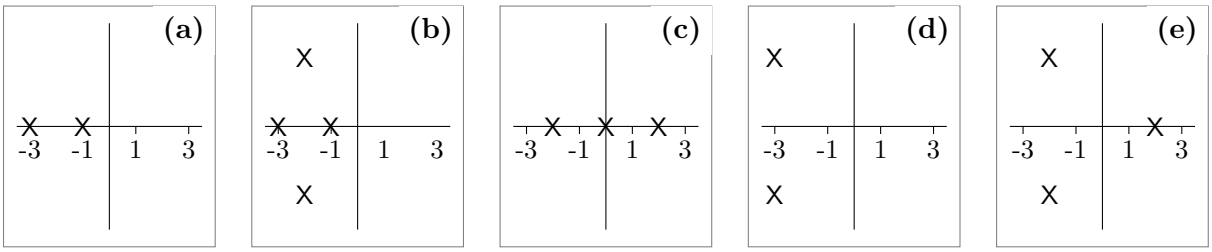
Problem 4. (10 points)
Express $(D + tI)(D - tI)$ in the form $a(t)D^2 + b(t)D + c(t)I$. (That is, your answer should have a form like $tD^2 + (t+1)^2D + (t-1)^2I$.)

(Hint, apply this operator to an arbitrary test function f.)

Over for Problem 5

Problem 5. (10 points)
Each of the pole diagrams below are in the complex plane and the crosses give the characteristic roots of an equation $P(D)x = 0$. (So $P(D)$ is different for each diagram.)

For each part of this problem, you must give a short explanation for your answer.



(a) (4) List the plot(s) which represent *stable* systems.

(b) (3) Plot (a) represents a (second-order) damped harmonic oscillator. Is the oscillator under, over or critically damped?

(c) (3) Which of the systems decays to 0 the fastest? You must give a short explanation.

End of quiz

ES.1803 Differential Equations

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