ES.1803 Problem Section Problems for Quiz 3, Spring 2024

Topic 9: Amplitude response, resonance and practical resonance

Problem 9.1. Consider the system $x'' + 8x = F_0 \cos(\omega t)$.

(a) Why is this called a driven undamped system?

(b) Solve this using the sinusoidal response formula (SRF). Then do it again using complex replacement and the exponential response formula (ERF).

(c) Consider the right hand side of the DE to be the input. Graph the amplitude response function.

(d) What is the resonant frequency of the system?

(e) Why is this called the natural frequency?

Problem 9.2. Consider the forced damped system: $x'' + 2x' + 9x = \cos(\omega t)$.

(a) What is the natural frequency of the system?

(b) Find the response of the system in amplitude-phase form.

(c) Consider the right hand side of the DE to be the input. What is the amplitude response of the system? Draw its graph –be sure to label your axes correctly

(d) What is the practical resonant frequency?

(e) When $\omega = \sqrt{7}$ by how many radians does the output peak lag behind the input peak?

Problem 9.3. Consider the system

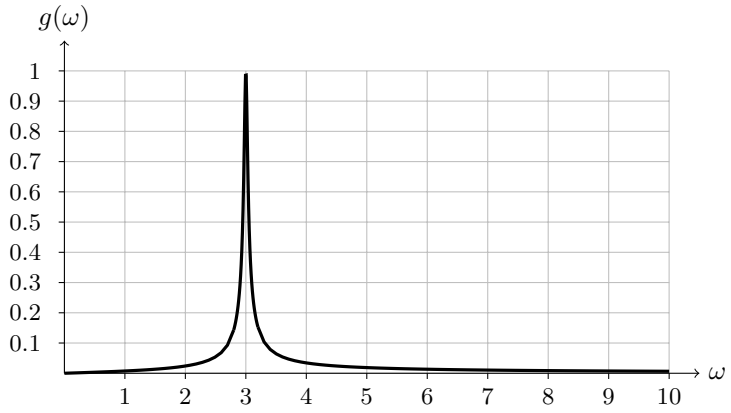
$$2y'' + 10y' + 3y = 3B\cos(\omega t),$$

where we consider $B\cos(\omega t)$ to be the input. Find and graph the gain. Find the practical resonant frequency.

Problem 9.4. For the forced undamped system $x'' + 8x = F_0 \cos(\omega t)$, give a detailed description of the phase lag for different input frequencies. (Consider $F_0 \cos(\omega t)$ to be the input.)

Problem 9.5. Consider the driven first-order system: $x' + kx = kF_0 \cos(\omega t)$. We'll take the input to be $F_0 \cos(\omega t)$. Solve the DE. Find the amplitude response. Show there is never practical resonance.

Problem 9.6. Below is a gain curve. Suppose the input is $\sum_{n=0}^{100} \cos(nt)$. Give a rough sketch of the output.


$g(\omega)$
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
1
2
3
4
5
6
7
8
9
10
ω

ES.1803 Differential Equations

Spring 2024