Name _____

ES.1803 Quiz 3, Spring 2024

4 problems, No books, notes or calculators.

Problem 1. (30 points)
Consider the system $2x'' + 5x' + 8x = \omega\cos(\omega t)$, where we consider $\cos(\omega t)$ to be the input.

(a) (15) Find the sinusoidal response, gain and phase lag of this system.

(b) (7) Find all the practical resonant frequencies of this system.

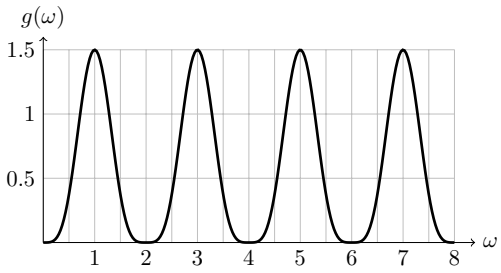
(c) (5) Plot the gain of this system.

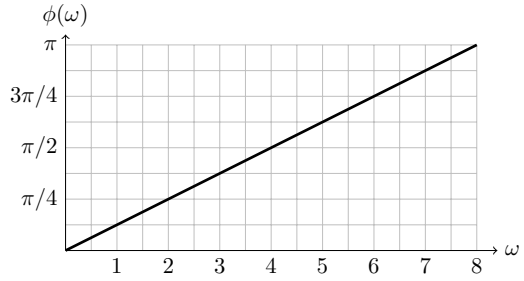
(d) (3) This system models a damped harmonic oscillator. Find the natural frequency of the oscillator.

Problem 2. (15 points)
(a) (10) Suppose we have a stable linear constant coefficient system $P(D)x = f$, where we consider f to be the input. The plots of gain $g(\omega)$ and phase lag $\phi(\omega)$ are shown below. Give the periodic solution to the following DE.

$$P(D)x = \cos(t) + \cos(2t) + \cos(3t).$$

You must give a brief explanation of your reasoning.





(b) (5) What are the resonant frequencies of this system?

Problem 3. (15 points)
(a) (10) Find one solution to the equation $x'' + 9x = \cos(3t)$.

(b) (5) Draw a graph of the solution from Part (a).

Problem 4. (10 points)
(a) (5) Consider the system $x'' + 5x' - 6x = f(t)$, where we consider $f(t)$ to be the input. Explain why we would not talk about the gain for this system.

(b) (5) Explain why a system $P(D)x = \cos(\omega t)$ that has a pure resonant frequency also has a sinusoidal solution to its homogeneous equation $P(D)x = 0$.

End of quiz

ES.1803 Differential Equations

Spring 2024