Differential Equations Review Sheet FirstOrderNonLinear, Spring 2024

II. Euler's Method: $y' = f(x, y)$

- What is it?

  Linear approximation of next value of solution, using $f(x, y)$ to get the slope of the tangent line at the current point.
- How do I calculate it? : $x_{\text{new}} = x_{\text{old}} + h, \quad y_{\text{new}} = y_{\text{old}} + h \cdot f(x_{\text{old}}, y_{\text{old}})$.
- Using indices: $x_{n+1} = x_n + h, \quad y_{n+1} = y_n + hf(x_n, y_n)$
- Was my approximation too high? too low?

  Look at concavity: Find y'' by differentiating $y' = f(x, y)$ WRT x (don't forget to use the chain rule and that $y' = f(x, y)$). Find the value of y'' at given point (x, y):

  if $y'' > 0$, then the solution is concave up, so the approximation is probably too low.

  if $y'' < 0$, then the solution is concave down, so the approximation is probably too high.

III Existence Uniqueness Theorem for $y' = f(x, y)$

- Statement of theorem:
  - If $f(x, y)$ is continuous on a rectangle which contains (a, b), then the IVP

    $$y' = f(x, y), \ y(a) = b$$

    has at least one solution on open interval J, containing the point a.
  - If $\frac{\partial f}{\partial y}$ is continuous on that rectangle, then the solution is UNIQUE on some open interval containing the point $x = a$.
- How to apply it
  - Check to see if $f(x, y)$ is continuous on an interval containing the point.
  - Check to see if $\frac{\partial f}{\partial y}$ is continuous on an interval containing the point.
  - If yes to both, then there's only ONE solutions through point, (x_0, y_0). So hah, in that case, no two solutions can cross (or even touch) because then they'll share that point.

IV Autonomous DEs of the form $x' = f(x)$

To get qualitative and long term information:

- Find critical points (equilibrium solutions): $f(x) = 0$.
- Find intervals where x is increasing or decreasing.
- Plot phase line, include critical points and stability.
- Sketch solutions in xt-plane.
- If there is a parameter, you can sketch the bifurcation diagram to show how the system behaves for different values of the parameter.

End of review sheet FirstOrderNonLinear

ES.1803 Differential Equations

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