Differential Equations Review Checklist, Spring 2024

1 Unit 1

Topic 1: Introduction; separable equations; modeling

- Two most important DEs
- Modeling
- Separable DEs
- Geometric problems
- IVP

Topic 2: Linear, first-order DEs

- First-order linear DEs
- Variation of params

Topic 3: Signals and systems

- Signals, input-response

Topic 4: Complex numbers

- Arithmetic
- Euler's formula
- Polar form
- nth roots
- Complex replacement

Topic 5: Homogeneous, constant coefficient, linear DEs $P(D)x = 0$

- Characteristic equation
- Real, repeated, complex roots
- Linearity/superposition
- Second-order systems: damped harmonic oscillators; over, under, critical damping
- Exponential decay rates
- Pole diagrams

Topic 6: Inhomogeneous, constant coefficient, linear DEs $P(D)x = f$

- ERF, SRF
- Linearity/superposition: $x = x_p + x_h$
- Complex replacement
- Driven damped harmonic oscillators
- Operators $P(D)$, T

Topic 7: Polymomial input; general operators

- Polynomial input: method of undetermined coefficients

- Algebra with CC and non-CC operators

Topic 8: Stability

- Stability of equilibria, physical (return to equilibrium)
- For linear CC equations: equilibrium at $x = 0$. Stable if $x_h \to 0$, i.e., real parts of roots are negative
- Routh-Hurwitz stability criteria

Topic 9: Engineering language

- Input, output
- (For stable systems) gain, phase lag, frequency response (formula and graphs)
- Pure and practical resonance
- Solving $P(D)x = f'$ using complex replacement

2 Unit 2: First-order nonlinear

Topic 10: Direction fields

- Integral curves
- Isoclines (& nullclines)
- Existence & Uniqueness Theorem
- Fences & funnels

Topic 11: Numerical methods

- General $y_{n+1} = y_n + m_n * h_n$ – choose m_n, h_n
- Euler's method

Topic 12: Autonomous equations

- Critical points
- Stability
- Phase lines
- Bifurcation diagrams

3 Unit 3: Linear Algebra

Topic 13: Vector spaces; matrices

- Vector spaces
- Matrix multiplication as linear combination of columns
- Linearity

Topic 14: Row reduction

- Row reduction, RREF
- Pivot, free variables, rank
- Subspaces, Null(A), Col(A) meaning and computation
- Solving $Ax = b$
- Span, independence, basis, dimension
- $\mathbf{R}^n$, standard basis

Topic 15: Determinants; inverses

- Inverse and determinants: By Laplace expansion and by row reduction
- Transpose
- $\det A = 0 \leftrightarrow \text{Null}A$ nontrivial
- Diagonal and triangular matrices

Topic 16: Eigenstuff

- Definition
- Characteristic equation, eigenspace=Null($A - \lambda I$)
- Diagonal and triangular matrices
- Diagonalization and decoupling

Topic 17: Matrix methods for systems of DEs

- Real, complex, repeated roots
- Companion systems
- Population models, coupled springs, mixing tanks

4 Unit 4: delta functions, Fourier and PDE

Topic 20: Delta functions

- Step and δ functions: $\delta(t) = u'(t)$
- Integrals
- Input to DEs: pre and post IC
- Generalized derivatives

Topic 21: Fourier series

- Definition, L = half period
- Terminology: period, fundamental frequency, harmonics, DC term
- Formula for coefficients

Topic 22: Fourier series continued

- Even and odd functions: simplified formulas for Fourier coefficients
- Sq(t): period 2π, odd, amplitude 1, square wave
- Tri(t): period 2π, even, triangle wave
- Decay rate of coefficients: jumps, corners, smooth

Topic 23: Tricks, sine and cosine series

- Calculation tricks for Fourier series: shifting, scaling, differentiating, integrating
- Fourier sine and cosine series for functions $f(x)$ on $[0, L]$

Topic 24: Periodic input

- $P(D)x = f(t)$, where $f(t)$ is periodic
- Pure and near resonance

Topic 25 and 26: PDEs, Fourier's method

- Heat and wave equation, boundary conditions
- Fourier method of separation of variables: modal solutions, general solution
- Initial conditions

5 Unit 5

Topic 27: Linear phase portraits

- Phase plane, trajectories, equilibrium at origin
- Main types of critical points at the origin
- Edge cases
- Dynamic stability
- Structural stability
- Trace-determinant diagram

Topic 28: Nonlinear autonomous systems

- Critical points, linearization, phase portraits, interpretation

Topic 29: Structural stability

- More details on structural stability

Topic 30: Population models

- Population models
- Volterra's principle

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