Massachusetts Institute of Technology

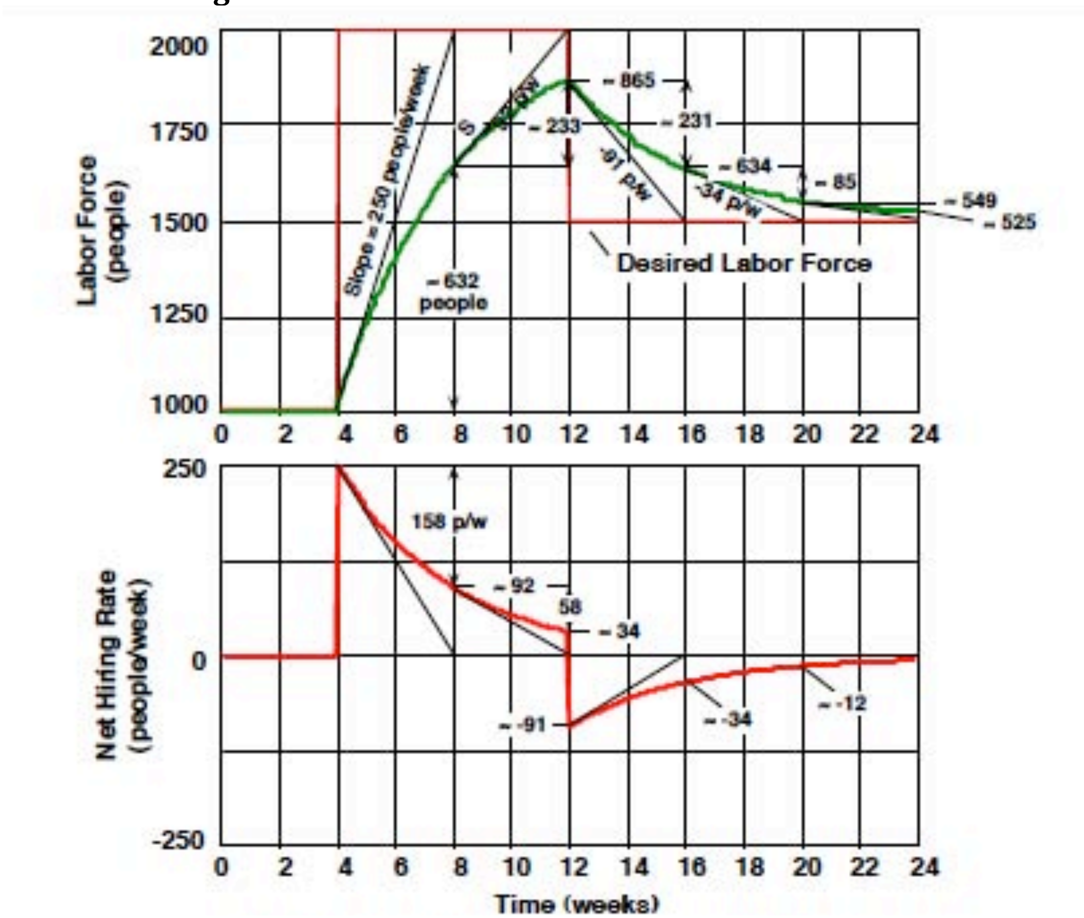
ESD.00 Introduction to Engineering Systems

Spring, 2011

Assignment # 3 Solutions

Assigned Date: Lecture 4
Due Date: Lecture 6

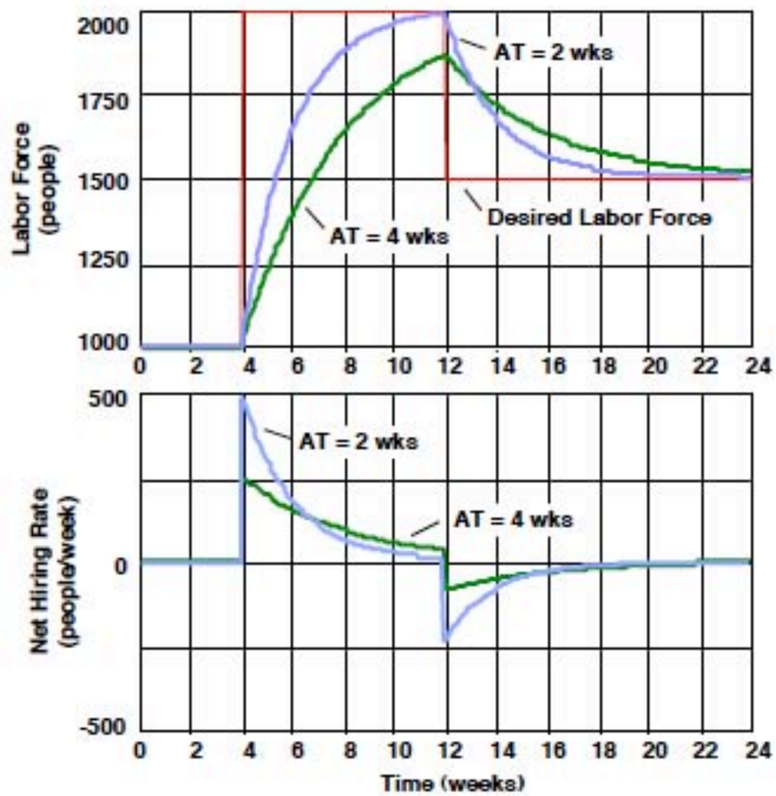
A. Goal-Seeking Behavior:



This is a first order system and the labor force will follow an exponential trajectory with a four-week time constant. At 4 weeks, the labor force will have covered 63% of the gap, so it'll be 1000+(1000*0.63) = 1630 people. At t = 8 weeks, i.e. after two time constants, it will be 87% of way there, therefore at ~ 1870 people and at 12 weeks (three time constants) it will be 95% at 1950.

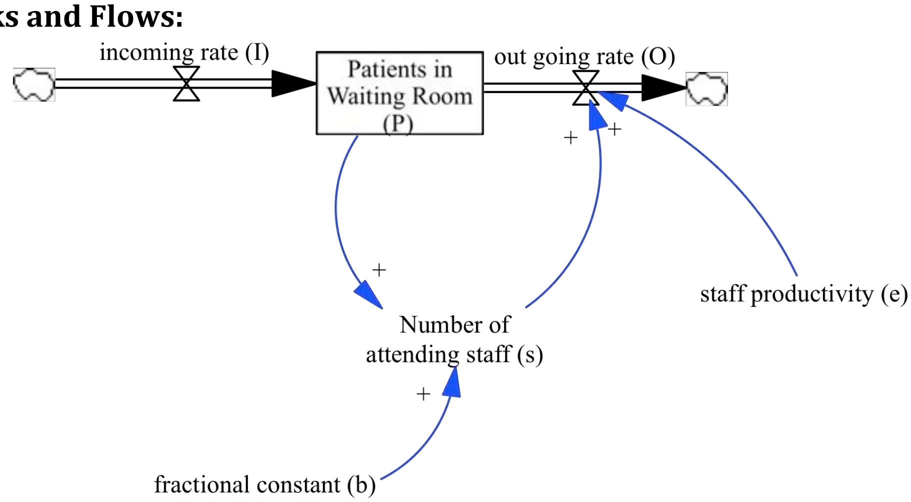
The desired number of workforce falls at t=12 to 1500. The gap is now 450. So again, the behavior will be exponential. And after 4 weeks (one time constant) at week 16 on the graph it will have covered 63% of the gap (450*0.63 = 283) and so on. By week 20 (8weeks after the change), it will be at 1950 - (450*0.87) = 1559.

With a two-week time constant, the response will be faster. The labor force will reach 98% of the gap (980 people) in 4 time constants, i.e. in 8 weeks. So with a step input at 4 weeks, with a gap of 1000 people, the labor force will reach 1980 in week 12. At week 12 there is another step (to 1500). The gap of 480 will decrease in 8 additional weeks (at week 20) to 98% of the desired level therefore it will be at 1980 – 470 = 1510.



(Reference: Business Dynamics, J. Sterman, 2000, Challenge 8-13)

B. Stocks and Flows:



The units are indicated in square brackets:

I: [patients/hour]

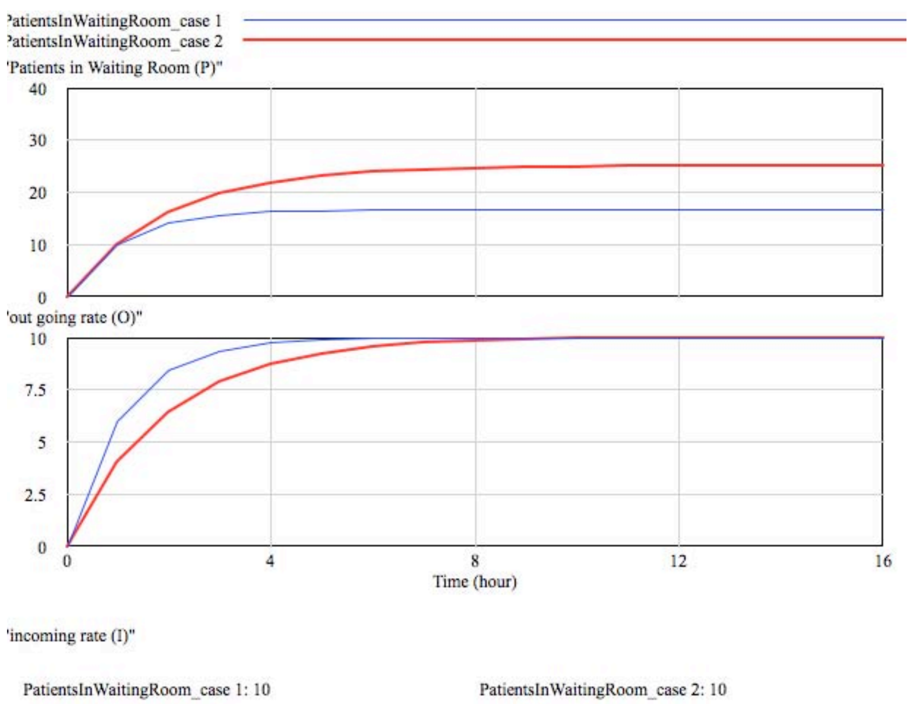
$P = P_o + \text{integral } [(I - O)\, dt]$, [patients]

$O = s*e$ [patients/hour]

$s = b*P$ [attendants]

b: [attendants/patients]

e: [patients/attendants/hour]



Time (hour)	"Patients in	"Patients in Waiting Room (P)"	
0	Waiting	0	0
1	Room (P)"	10	10
2	Runs:	14	16
3	PatientsIn	15.6	19.6
4	Waiting	16.24	21.76
5	Room_case	16.496	23.056
6	1	16.5984	23.8336
7	PatientsIn	16.6394	24.3002
8	Waiting	16.6557	24.5801
9	Room_case	16.6623	24.7481
10	2	16.6649	24.8488
11		16.666	24.9093
12		16.6664	24.9456
13		16.6666	24.9673
14		16.6666	24.9804
15		16.6666	24.9882
16		16.6667	24.9929

In the first (base) case, at the end of 2 hours, there are 14 people in the waiting room. By the end of 4 hours, there are about ~16 people in the room. By this time the system is mostly in steady state.

If the efficiency, e, is changed (so that each patient can get more time and more careful processing) to 4, then case_2 shown in blue in the graphs occurs. In this case, the number of people waiting at 2 hours is 16, and at 4 hours is 22. In steady-state there are 25 people waiting.

<u>Additional Explanation (Not required as part of solution):</u>

The system can be mathematically represented as:

$$\frac{dp}{dt} = I(t) - O(t)$$

$$O(t) = k_1 p \quad , k_1 = be$$

$$\Rightarrow \frac{dp}{dt} = I(t) - k_1 p$$

$$\frac{dp}{dt} + k_1 p = I(t)$$

This is a first order equation with an input I(t).

In our case, the input I(t) is constant. Let's denote it as k_2:

$$\frac{dp}{dt} + k_1 p = k_2$$

We can separate the variables p and t and then integrate:

$$\frac{dp}{k_2 - k_1 p} = dt$$

$$\Rightarrow \frac{dp}{p - k_2/k_1} = -k_1 dt$$

$$\Rightarrow \ln\left|p - \frac{k_2}{k_1}\right| = -k_1 t + c$$

$$\Rightarrow p - \frac{k_2}{k_1} = c_1 e^{-k_1 t}$$

$$p = \frac{k_2}{k_1} + c_1 e^{-k_1 t}$$

for $t_0 = 0$ and $p(0) = p_o$:

$$p_0 = \frac{k_2}{k_1} + c_1$$

$$\Rightarrow c_1 = p_0 - \frac{k_2}{k_1}$$

Substituting this value for c_1 in equation for p:

$$\boxed{p = \frac{k_2}{k_1} + \left(p_0 - \frac{k_2}{k_1}\right) e^{-k_1 t}}$$

Note that for large t, the value of p approaches the constant k2/k1.

For b= 0.1, e = 6, and $k_2 = 10$, we get:

$$p = \frac{k_2}{k_1} = \frac{k_2}{be} = \frac{10}{0.1 \times 6} = 16.67.$$

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