



# Introduction to Engineering Systems, ESD.00

Lecture 6

Lecturers:

Professor Joseph Sussman

Dr. Afreen Siddiqi

TA: Regina Clewlow



Massachusetts Institute of Technology  
**Engineering Systems Division**



# Uncertainty Lecture 2-- Outline

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Global Climate Change

High-impact, Low-probability events

Decision-making Under Uncertainty

    Lotteries

    Annuities

Compound Probabilities

    With Independence

    Conditional Probabilities

        Tsunami Example

    Bayes Theorem

        Examples--Snow Day at MIT, the Birthday Trick

Decision Trees

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# Uncertainty: Global Climate Change

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## Global Climate Change

People disagree, but everyone agrees there is a lot of uncertainty

Let's think about the kinds of uncertainty and how we could decide what to do

# Uncertainty: Global Climate Change

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## Global Climate Change

What kinds of uncertainties are there?

What should the purposes of our mitigation strategies be?

# Uncertainty: Global Climate Change

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## Global Climate Change Uncertainties

The rise in temperatures

The impact of a given rise in  
temperature

# Uncertainty: Global Climate Change

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## Global Climate Change

You could argue what you want to reduce is not necessarily the mean value of temperature rise, but rather the probability

of a temperature  $>$  some critical threshold

So develop strategies intended to reduce the right-hand tail of the pdf

Then look at impacts of various temperature rises

# Uncertainty: Global Climate Change

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How does feedback fit into this picture?

Positive?

Negative?

So we combine the first two thrusts of ESD.00

**UNCERTAINTY AND FEEDBACK**

# Uncertainty: High-impact, Low Probability Events

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Ripped from the headlines

The Japan earthquake and tsunamis

5th biggest earthquake in recorded history,  
biggest ever in Japan

Huge loss of life, injuries, property damage

Japan likely the most prepared nation in the  
world for earthquake disasters

# Uncertainty: High-impact, Low Probability Events

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Very high-impact, Very low-probability events  
Example--meteor strikes the earth

What should/can we do about that? It could be an extinction event

# Uncertainty: High-impact, Low Probability Events

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Tsunami on the East Coast of the U. S.?

Discussed in an article in the Sunday Boston Globe yesterday

# Uncertainty: High-impact, Low Probability Events

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But high-impact, low-probability events are only a special case of a broader topic:

## Decision-making under uncertainty

# Decision-making under uncertainty

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Decision-making under uncertainty

Lottery

I give you a choice:

\$10

or nothing with probability = .5 and

\$20 with probability = .5

What do you pick?

# Decision-making under uncertainty

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Decision-making under uncertainty

Lottery

I give you a choice:

\$10,000

or nothing with probability = .5 and

\$20,000 with probability = .5

What do you pick?

The concept of utility-- for most people, it's non-linear and it's asymmetric

What would the probability of \$20,000 have to be for you to accept the lottery and not the \$10,000 with certainty?

# Uncertainty: Annuities

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## Annuities

Buy an annuity for  $\$X$

You get  $\$Y$ / year for the rest of your life....

Why it is a [good, bad] deal for you?

Why it is a [good, bad] deal for the company that sold you're the annuity?

What might you do instead of buying an annuity?

# Uncertainty: Compound probabilities

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Assume independence

\_\_\_\_\_ A \_\_\_\_\_ x \_\_\_\_\_ B \_\_\_\_\_

Electrical example:  $P(A)$  is probability that A is conducting current; same for  $P(B)$

What is  $P(\text{current flows})$ :  $P(A) * P(B)$

# Uncertainty: Compound probabilities

Assume independence: A and B in parallel



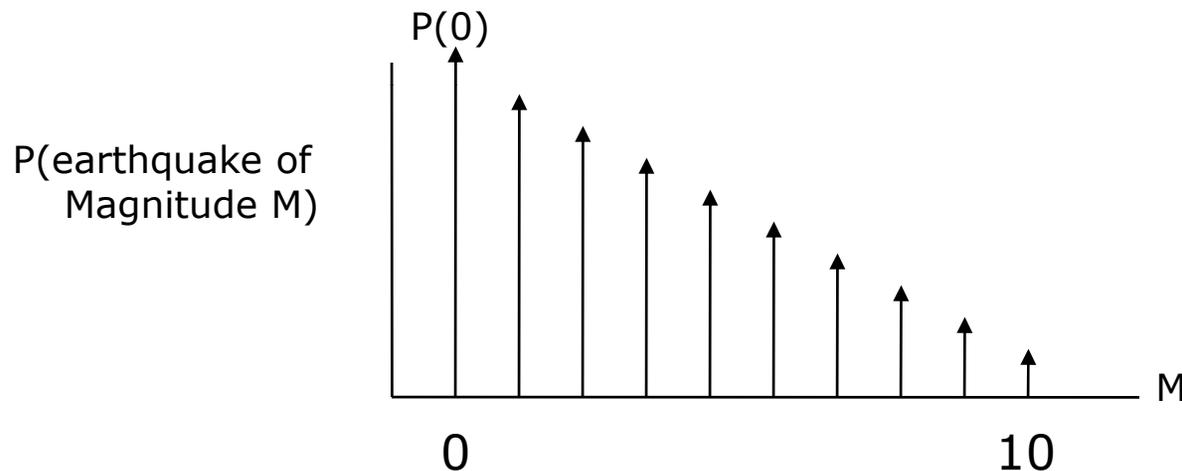
$$\begin{aligned} P(\text{ current flows}) &= P(\text{at least one link is conducting}) = \\ &1 - P(\text{neither A nor B conducting}) = \\ &1 - [(1 - P(A)) * (1 - P(B))] = P(A) + P(B) - P(A) * P(B) \end{aligned}$$

# Uncertainty: Compound conditional probabilities

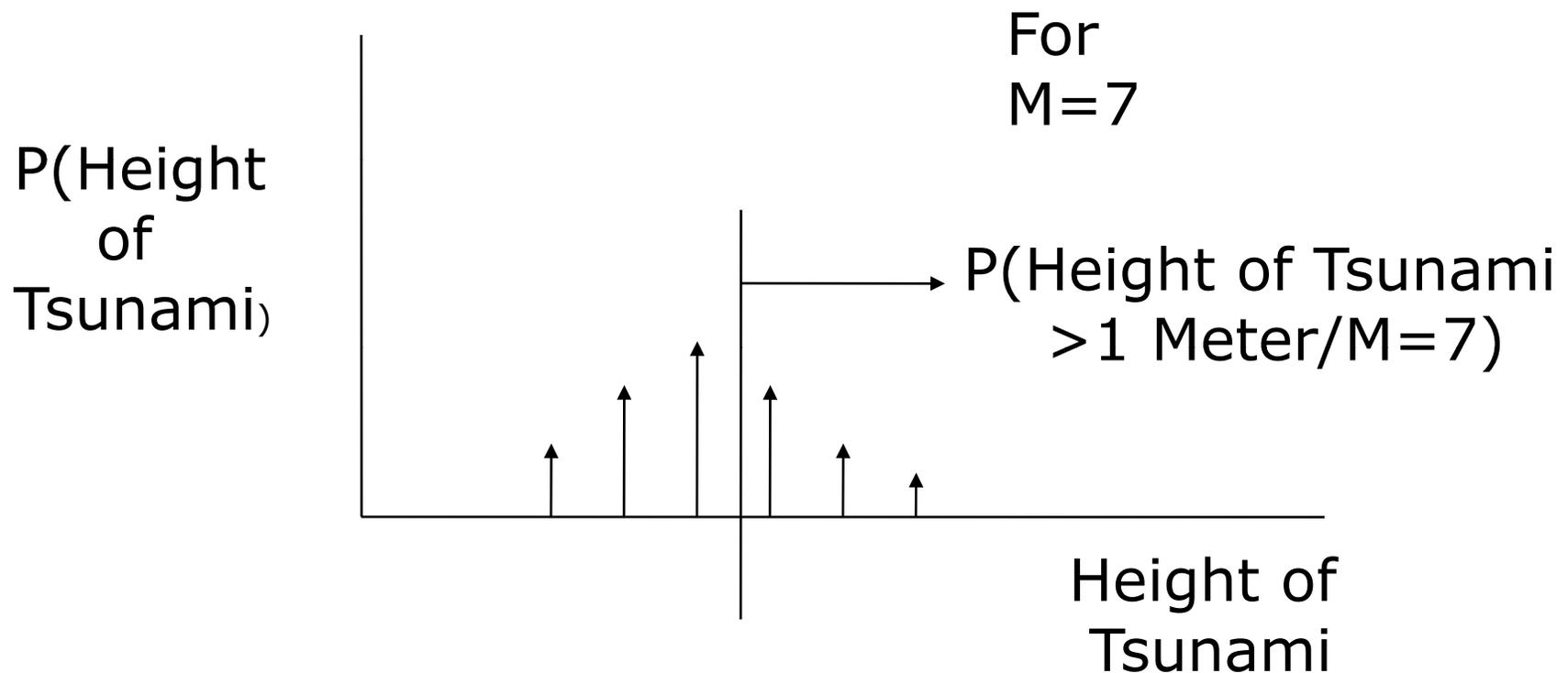
But, events may not be independent but rather conditional

What is the daily probability of a tsunami > 1 meter in Hawaii with its origins from an earthquake in Japan?

Simplifying assumption: assume at most 1 earthquake per day in Japan



# Uncertainty: Compound conditional probabilities



# Uncertainty: Compound probabilities

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$P(\text{tsunami} > 1 \text{ meter})$

$$\sum_{\text{All } M} P(\text{tsunami} > 1 \text{ meter}/M) P(M)$$

# Uncertainty: Bayes' Theorem

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## Bayes' Theorem

### Conditional probabilities

$P(\text{event A happens}) = [P(\text{event A/given B occurs}) \text{ for all possible outcomes of B}] * P(\text{each possible outcome of B})$

# Uncertainty: Bayes' Theorem

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The MIT Snow Day example

# Uncertainty: Bayes' Theorem

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The birthday example: How many birthdays until a match?

# More on Decision-making Under Uncertainty:

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Decision-making under uncertainty  
Decision trees:

Example: Football

See teaching note  
“Did Belichick Make  
the Right Call?”

# More on Decision-making Under Uncertainty:

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Decision-making under uncertainty

Decision trees:

Implicit Assumption: All Belichick cares about is winning – this is likely true

But what about another coach of lesser stature?

Consider the “embarrassment” factor!

See teaching note “Belichick –Part 2”

# More on Decision-making Under Uncertainty:

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If you don't consider the “embarrassment” factor, a decision may appear to the outside observer to be “irrational”.

Example: “Why in the world don’t more people take public transportation to work when it is faster and less expensive than driving? People are just irrational!”

We looking from the outside may not understand other factors the traveler considers to be important.

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ESD.00 Introduction to Engineering Systems  
Spring 2011

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