Analysis and Synthesis:

A] Anonymous scholium to Euclid XIII:

“Analysis, then, is the taking of what is sought as if it were agreed to be true and following through the consequences from this to something genuinely agreed to be true, while synthesis is the taking of something admitted and going through to something we then agree to be true.” [I took some liberties with translation to make this clearer to you. Apologies to Anonymous]

B] Pappus of Alexandria [neo-Platonist, 4th century A.D.):

“... in analysis we suppose that which is sought to be already done, and inquire what it is from which this comes about, and again what is the antecedent cause of the latter, and so on until, by retracing our steps, we light upon something already known or ranking as a first principle... But in synthesis, proceeding in the opposite way, we suppose to be already done that which was last reached in analysis, and arranging in their natural order as consequents what were formerly antecedents and linking them one with another, we finally arrive at the construction of what was sought...”

“Analysis, then, is the way from what is sought, taken as admitted by means of previous synthesis...but in synthesis, going in reverse, we suppose as admitted what was the last result of the analysis, and, arranging in their natural order as consequences what were formerly the antecedents, and connecting them with one another, we arrive at the completion of the construction of what was sought, and call it synthesis.”

C] Francois Viete (the true founder of Western algebra, 16th century): “In mathematics there is a certain way of seeking the truth, a way which Plato is said first to have discovered, and which was called ‘analysis’ by Theon and was defined by him as ‘taking the thing sought as granted and proceeding by means of what follows to a truth that is uncontested’; so, on the other hand, ‘synthesis’, is ‘taking the thing that is granted and proceeding by means of what follow to the conclusion and comprehension of the thing sought

D] Rene Descartes (16th century) in Objections and Replies:

“...the method of proof is two-fold, one being analytic, the other synthetic. Analysis shows the true way by which a thing was methodically discovered and derived, as it were effect from cause, so that, if the reader care to follow it and give sufficient attention to everything, he understands the matter no less perfectly and makes it as much his own as if he had himself discovered it. But it contains nothing to incite belief in an inattentive or hostile reader; for if the very least thing brought forward escapes his notice, the necessity of the conclusions is lost; and on many matters which, nevertheless, should be specially noted it often scarcely touches, because they are clear to anyone who gives sufficient attention to them.

“Synthesis contrariwise employs an opposite procedure, one in which the search goes as it were from effect to cause (though often here the proof itself is from cause to effect to a greater extent than in the former case). It does indeed clearly demonstrate its conclusions, and it employs a
long series of definitions, postulates, axioms, theorems and problems, so that if one of the conclusions that follow is denied, it may at once be shown to be contained in what has gone before. Thus the reader, however hostile and obstinate, is compelled to render his assent. Yet this method is not as satisfactory as the other and does not equally well content the eager learner, because it does not show the way in which the matter taught was discovered.

“It was synthesis alone that the ancient Geometers employed in their writings, not because they were wholly ignorant of the analytic method, but, in my opinion, because they set so high a value on it that they wished to keep it to themselves as an important secret.”