Medical Image Registration II

HST 6.555

Lilla Zöllei and William Wells

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CT-MR registration movie

Roadmap

✓ Data representation
✓ Transformation types
✓ Objective functions
  ✓ Feature/surface-based
  ✓ Intensity-based
⚠ Optimization methods
■ Current research topics
Medical Image Registration

Medical image data sets

Transform (move around)

Compare with objective function

Optimization algorithm

motion parameters

score

initial value
find $x$ that minimizes $f(x)$

convex:

- single minimum
- global minimum

non-convex:

- local minima
- capture region
- global minimum
Goal: find $x$ that optimizes $f(x)$

- do it quickly, cheaply, in small memory; (or evaluate $f$ as few times as possible)

Parameter recovery: “search” for solution

- Standard mathematical function (with T dependency) to be optimized
  - use only function evaluations
  - use gradient calculations (more guidance, but costly)

Based upon prior information:

- *constrained*, e.g.: $x_1 \leq x \leq x_2$
- *unconstrained*
No guarantees about global extremum

- Local extrema:
  - sometimes sufficient**
  - find local extrema from a wide variety of starting points; choose the best
  - perturb local extremum and see whether we return

- Ambitious algorithms:
  - simulated annealing methods
  - genetic algorithms
Search Algorithms

- **1D solutions** – minimum bracketing is possible
  - Golden Section Search
  - Brent’s Method
  - Steepest Descent

- **Multi-dimension** – initial guess is important!
  - Downhill Simplex (Nelder & Mead)
  - Direction Set Methods
    - Coordinate Descent
    - Powell’s Method
  - Gradient Methods
    - Conjugate gradient methods
Root finding by bisection

- Root-bracketing by two points: \((P_1, P_2)\)

\[
\text{sgn}(f(P_1)) \neq \text{sgn}(f(P_2))
\]
Minimization of convex function

- Minimum bracketing in \((P_1, P_2)\):
  \[
  P_1 < P_3 < P_2 \\
  f(P_3) < f(P_1) \\
  f(P_3) < f(P_2)
  \]

\(P_3\): best current estimate of the location of the minimum
Golden Section Search

Minimization strategy:

(i) select the larger of \( \frac{P_1P_3}{P_3P_2} \) → assign this interval to be \( L \)

(ii) position \( x \) in \( L \) s.t.
\[
\frac{\| \overrightarrow{P_3x} \|}{\| L \|} = \frac{3 - \sqrt{5}}{2} \approx 0.38197*
\]

(iii) new bracketing triplet is:
\( (P_1, x, P_3) \) if 
\[ f(x) < f(P_1) \text{ and } f(x) < f(P_2) \]
\( (x, P_3, P_2) \) if 
\[ f(P_3) < f(x) \text{ and } f(P_3) < f(P_2) \]

*golden mean / golden section (Pythagoreans)
Brent’s Method

- Minimum bracketing
- Parabolic interpolation
Gradient Descent

\[ m = \text{slope} = \frac{df(x)}{dx} \]

\[ P' = P + \alpha m \]
Downhill Simplex Method (1)

- due to Nelder and Mead*
- self-contained; no 1D line minimization
- only function evaluations, no derivatives
- not efficient in terms of number of function evaluations, but easy-to-implement
- geometrical naturalness
- **useful:** when $f$ is non-smooth or when derivatives are impossible to find

Downhill Simplex Method (2)

- **simplex**: geometrical figure; in N dimensions, (N+1) points/vertices
  - e.g.: in 2D: triangle, in 3D: tetrahedron
  - non-degenerate! (encloses a finite N-dimensional volume)

- starting guess ((N+1) points)
  - $\mathbf{P}_0$ and $\mathbf{P}_i = \mathbf{P}_0 + \lambda \mathbf{e}_i$, $\mathbf{e}_i$: unit vectors;
    $\lambda$: constant, guess of characteristic length scale
animation of progress of sequential simplex
Valid simplex steps:

- (a) reflection
- (b) reflection and expansion
- (c) contraction
- (d) multiple contraction
possible moves (from previous figure):
- reflection (conserving volume of the simplex)
- reflection and expansion
- contraction
- multiple contraction

termination criterion
- use threshold on moved vector distance
- or threshold on function value change

restart strategy
- needed as even a single anomalous step can fool the search algorithm
Implementation details

- `fminsearch` in MATLAB
  1. build initial simplex
  2. do reflections, expand if appropriate
  3. in “valley floor” contract transverse
     - ooze down valley

- works well in some medical registration methods
- has implicit *coarse-to-fine* behavior
animation of progress of sequential simplex
Direction Set Methods

- Successive line minimizations
- No explicit gradient calculation
- How to select the best set of directions to follow?
  - simple example: follow the coordinate directions
  - direction set methods: compute “good” or non-interfering (conjugate) directions
Follow the coordinate directions

contours of $f(x,y)$

Ideal situation: only two steps are enough to locate the minimum
In general: can be very inefficient; large number of steps can be required to find the minimum.
Conjugate Directions

- non-interfering directions: subsequent minimizations should not spoil previous optimization results

- goal: come up with a set of N linearly independent, mutually conjugate directions

⇒ N line minimizations will achieve the minimum of a quadratic form
Powell’s Method

- N(N+1) line minimizations to achieve the minimum
- possible problem with linear dependence after update
  - fix
    - re-initialize the set of directions to the basis vectors
    - few good directions (instead of N conjugate ones)
Conjugate Gradient Methods

- gradient calculation is needed
- order N separate line minimizations
- computational speed improvement
  - Steepest Descent method
    - right angle turns at all times
  - Conjugate Direction methods
Steepest descent method – still a large number of steps is required to find the minimum.

See http://www.tcm.phy.cam.ac.uk/~pdh1001/thesis/node57.html
Conjugate gradients method - only two steps are required to find the minimum.

See http://www.tcm.phy.cam.ac.uk/~pdh1001/thesis/node57.html
Simulated Annealing

- exploits an analogy between the way in which a metal cools and freezes into a minimum energy crystalline structure (the annealing process) and the search for a minimum in a more general system

- employs a random search accepting (with a given probability) both changes that decrease and increase the objective function

- successful at finding global optima among a large numbers of undesired local extrema
Genetic Algorithm

- works very well on mixed (continuous and discrete), combinatorial problems; less susceptible to getting 'stuck' at local optima than gradient search methods
- tends to be computationally expensive
- represents solution to the problem as a genome (or chromosome); creates a population of solutions and apply genetic operators (mutation, crossover) to evolve the solutions in order to find the best one(s).
- most important aspects of using genetic algorithms are
  - (1) definition of the objective function
  - (2) definition and implementation of the genetic representation
  - (3) definition and implementation of the genetic operators
- [http://lancet.mit.edu/~mbwall/presentations/IntroToGAs/](http://lancet.mit.edu/~mbwall/presentations/IntroToGAs/)
Coarse-to-Fine Strategy

- **Technique:**
  - smooth objective function $f_N$ (e.g.: blur with Gaussian)
  - optimize smoothed version (use result as start value for original objective $f_N$)

- **Advantages:**
  - avoiding local extrema
  - speed up computations
$f$ really smoothed

$f$ smoothed

search

start

search

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More examples

- *Numerical Recipes in “C”*
  [http://www.nrbook.com/nr3/](http://www.nrbook.com/nr3/)
Current Research topics

- group-wise (vs. pair-wise) registration
- Diffusion Tensor (DT) MRI alignment
- surface-based (vs volumetric) alignment

- Open questions: tumor growth modeling, structural – functional alignment, ….
- Registration evaluation and validation
Group-wise registration

From Lilla Zöllei’s thesis research.

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Group-wise registration styles

- Template-dependent
  - Fixed template
    - Arbitrary member of the population
    - Pre-defined atlas
  - Online computed template
    - Sequential pair-wise alignment to evolving “mean”

- Template-free
  - Simultaneous
The Congealing method

- **Def.**: simultaneous alignment of each of a set of images to each other

- Applications:
  - Handwritten digit recognition (binary data)
  - Preliminary baby brain registration (binary data)
  - Bias removal from MRI images
Advantages of Congealing

- Computational advantages
- Can accommodate very large data sets
- Can accommodate multi-modal data
- Robust to noise and imaging artifacts
- No single central tendency assumption
Adult brain data set - mean volumes

**Unaligned** input data sets

**Aligned** input data sets

Data set: 28 T1-weighted MRI; [256x256x124] with (.9375, .9375, 1.5) mm³ voxels

Experiment: 2 levels; 12-param. affine; N = 2500; iter = 150; time = 1209 sec!!

Central coronal slices

Central coronal slices

Variance volume - during registration
Baby brain data set – central slices


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Baby brain data set – central slices


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Very large data set


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Affine + B-splines Deformation

Courtesy of Serdar K Balci and Kinh Tieu. Used with permission.
DT MRI alignment

Figure 2 from this article (sequence of eight images) removed due to copyright restrictions.

Surface-based alignment

Combined surface-based and volumetric alignment

Images removed due to copyright restrictions.

Courtesy of Gheorghe Postelnicu. Used with permission.
Further open questions:

- tumor growth modeling
- structural – functional alignment (MRI-fMRI)
- population comparison
- ....
Registration evaluation and validation

- Retrospective Image Registration Evaluation Project (Vanderbilt University, Nashville, TN)
  http://www.vuse.vanderbilt.edu/~image/registration/

- Non-Rigid Image Registration Evaluation Program (NIREP); University of Iowa
  http://www.nirep.org