Systems of pitch relations:
Scales, tunings and notations
What are scales, and why do we have them?

• Systems of discrete pitches & intervals between them
• Determine tonal systems
• Wide variety of scales in the world
• Octaves, fifths, & fourths very common
• Numbers of notes:
  – pentatonic (5 notes/octave)
  – diatonic (7 notes/octave)
  – chromatic (12 notes/octave)
  – "microtonal" (from Western perspective, > 12 notes/octave)
  – Arab music (24-TET), 31-TET
• Possible reasons for numbers of notes (7±2)
  – although musicians & listeners easily keep track of more
Scales: which intervals are included, sequence of intervals

- **Scales have reference points** (first note in a scale)
  - tonic, key, frame of reference, often first note played
  - action of auditory short-term memory & relative pitch
- Ordering of musical intervals within the scale
- Modes (Greek or ecclesiastical modes)
- **Major scale** pattern T-T-S-T-T-T-S (T whole tone, 12%, S semitone 6%)
- **Minor scale** pattern: T-S-T-T-S-T-T
  - Note same sets of intervals in related major & minor scales that use
    same notes (C major, A minor), and interval sets but w. different interval
    relationships to the tonic
- Circle of fifths: which scales share the same notes,
  - therefore have many common harmonics & subharmonics
  - and are therefore related in pitch space; distance reflects difference
- Ascending vs. descending intervals
- Scale notes vs. expressive deviations from them (“ornaments”)
Note similarity/compatibility and contrast as a basis for scale

Tonal hierarchy of similarity relations

Alphabet = 12 pitches of chromatic scale
C C# D D# E F F# G G# A A# B

Scales = subgroups of 7 pitches
24 Scales: 12 Major + 12 Minor

Intra-key hierarchies

Inter-key hierarchies

For Harmony

I
V
IV
VI
II
III
VII

Circle of fifths

Figure by MIT OpenCourseWare.
Tuning systems and scales

We will discuss tuning systems first, then scales, but the two are interrelated and affect each other.
Design constraints: What intervals to include? How many notes in the scale?

- Presence of human voices (tendency for just intervals)
- Contrast: similar and dissimilar notes (re: tonic)
- Small numbers of notes (recognizability)
- Choice of note combinations: consonance, roughness
- Musical instruments available
Design constraints: What intervals to include? How many notes in the scale?

Musical instruments and scale systems

Make specialized tunings possible
Technology to implement tunings (logarithms)
Versatility, compatibility w. other instruments
Ability to sound reasonably good in all keys
Equal temperament (log ratios)

(beginning late 16th with Vincenzo. Galileo, Zhu Zaiyu, China)

Non-equal temperament produces “colors” or “moods” for different keys (for better or worse)
Pythagorean ratios

Images removed due to copyright restrictions.

Figure 2-5. Greek citharis. The cithara was sacred to Apollo.
Pierce, *The Science of Musical Sound*
Mathematics, numerology, music, and mysticism

Drawn by Robert Fludd, the **Monochord** invented by Pythagoras, showing correspondences between pitch, proportion and astral bodies. [http://www.elodielauten.net/concept.html](http://www.elodielauten.net/concept.html)
Monochords & tuning

The monochord consisted of a single string stretched over a sound box, with the strings held taut by pegs or weights on either end. It used a moveable bridge to change pitch, and was usually plucked. A later instrument of the same principle was played with a bow, called the "trumpet marine" (Adkins, New Grove). It was used as an instrument as early 300 BC by Euclid (Ripin, New Grove), and as a scientific instrument by Pythagoras as early as the 6th century BC. No one knows when it first appeared, as its origins extend into prehistory.

Courtesy of Jeff Cottrell. Used with permission.
Scales begin with musical practice -- what sounds best is the criterion for a good scale does it give the composer a beautiful palette of notes?

11th century: Guido of Arezzo (fl. ~991/2-1033). "By laying out the notes of a scale on a monochord, he was able to teach choir boys how to sing chant and also to detect incorrect chanting. The tones intervals used in chants were M2, m2, m3, P4, and P5. These six intervals were the "consonantiae", and (according to Guido) no chant uses any other intervals. Guido admits one could find more on the monochord if "art (did not) restrain us by its authority" (Palisca)." Cottrell, History of the Monochord

Images removed due to copyright restrictions.
Note similarity/compatibility as a basis for scale

Include just ratios (voices)

Minimize clashing of tones, i.e. roughness
Max fusion
(Sethares: optimize spectrum for tunings to min beating e.g. 10-TET)

Provide contrast between consonant & dissonant relations

Systematicity: allow for chord/key modulations

K & K psychophysical data (circles)

Image removed due to copyright restrictions.

Some history of tuning systems (consult Wikipedia, many books for more)

Just tuning -- ancient & folk traditions, integer ratios

Pythagorean tuning -- pre-Renaissance, fifths & octaves

Meantone temperament -- Renaissance, tempered fifths

Well-temperament - Baroque, “moods & colors” of keys
Some keys more out of tune, Bach, Tartini advocates

Equal-temperament (12-TET)

Common Practice Period of Western music (~1600-1900)
Vincenzo Galileo (1581)
Ancient China (7-TET), and Zhu, Zaiyu (12-TET, 1584)
Early tuning systems

Pythagorean & Just tuning systems

Figure 10.2

Musical scale construction based on simple numerical ratios. Pythagorean scale notes are generated by using the consonant 3:2 ratio. Each new note is generated by multiplying the previous frequency by 3/2. In the top panel, the solid arrows show how the generated notes, and the dashed arrows show the octave equivalents necessary to keep the scale notes within a single octave. As shown below the notes, the frequency ratio between adjacent notes is not the same across all possible pairs. Just tuning notes are generated using small whole number ratios in the bottom panel. Difficulties arise when fitting in extra notes. There are two possibilities shown for the second note. Other notes (some sharps and flats) also cannot be defined unambiguously. The intervals beginning will be first note of the scale and intervals beginning with different notes of the scale are shown.

Courtesy of MIT Press. Used with permission.
Some history of tuning systems (consult Wikipedia, many books for more)

Twelve tone equal temperament took hold for a variety of reasons.

It conveniently fit the existing keyboard design, and was a better approximation to just intonation than the nearby alternative equal temperaments.

It permitted total harmonic freedom at the expense of just a little purity in every interval.

This allowed greater expression through enharmonic modulation, [a means of shifting between keys using chords that are shared by two keys].... Equal temperament became the standard gradually during the Romantic era.
Designing a scale system - 12 equally tempered notes/oct.
Problems that tuning creates and solves.... (Wikipedia, Musical temperament)

"Just intonation has the problem that it cannot modulate to a different key (a very common means of expression throughout the Common practice period of music) without discarding many of the tones used in the previous key, thus for every key the musician wishes to modulate to, the instrument must provide a few more strings, frets, or holes for him or her to use. When building an instrument, this can be very impractical.

Well temperament is the name given to a variety of different systems of temperament that were employed to solve this problem, in which some keys are more in tune than others, but all can be used. This phenomenon gives rise to infinite shades of key-colors, which are lost in the modern standard version: 12 tone equal temperament (12-TET). Unlike Meantone temperament, which alters the fifth to temper out the Syntonic comma, 12-TET tempers out the Pythagorean comma, thus creating a cycle of fifths that repeats itself exactly after 12 steps. This allowed the intervals of Tertian harmony, thirds and fifths, to be fairly close to their just counterparts (the fifths almost imperceptibly beating, the thirds a little milder than the Syntonic beating of Pythagorean tuning), while permitting the freedom to modulate to any key and by various means (e.g. common-tone and enharmonic modulation, see modulation). This freedom of modulation also allowed substantial use of more distant harmonic relationships, such as the Neapolitan chord, which became very important to Romantic composers in the 19th century."
C-Major diatonic scale

Figure by MIT OpenCourseWare.
Estimated consonance of pure & complex tone dyads

Kameoka & Kureagawa psychophysical data

Temporal pitch multiplicity model

Pure tones
Complex tones

Diatonic scale

Figure by MIT OpenCourseWare.
Diatonic scale

"In Music theory, the diatonic major scale is a fundamental building block of the Western musical tradition. The diatonic scale is composed of two tetrachords separated by intervals of a whole tone. The pattern of intervals in semitones is as follows 2-2-1-2-2-2-1. The major scale begins on the first note and proceeds by steps to the first octave. In solfege, the syllables for each scale degree are "Do-Re-Mi-Fa-Sol-La-Ti-Do". The natural minor scale can be thought of in two ways, the first is as the relative minor of the major scale, beginning on the sixth degree of the scale and proceeding step by step through the same tetrachords to the first octave of the sixth degree. In solfege "La-Ti-Do-Re-Mi-Fa-Sol." Alternately, the natural minor can be seen as a composite of two different tetrachords of the pattern 2-1-2-2-1-2-2. In solfege "Do-Re-Mi-Fa-Sol-La-Ti-Do." All of non-folk Western harmony from the some point in the late Renaissance up to the late nineteenth century is based upon these two objects and the unique relationships created by this system of organizing 7 notes. It should be kept in mind that most pieces of music change key, and thus scale, but are still related to the beginning diatonic scale. The white keys on a piano correspond to the diatonic scale of C major (C-D-E-F-G-A-B-C), with the notes a whole tone apart, except for E-F and B-C, which is an interval of a semitone (half a tone). Diatonic comes from the greek "diatonikos" or "to stretch out". It is sometimes used to refer to all the modes, but is generally used only in reference to the major and minor scales."

-- Wikipedia
# Three tuning systems

<table>
<thead>
<tr>
<th>Interval name</th>
<th>Solfeggio</th>
<th>Letter notation</th>
<th>Pythagorean tuning (PT)</th>
<th>Just intonation (JI)</th>
<th>Equal temperament (ET)</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Numerical origin</td>
<td>Frequency ratio</td>
<td>Cents</td>
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<tr>
<td>Unison</td>
<td>DO</td>
<td>C</td>
<td>1:1</td>
<td>1.000</td>
<td>0.0</td>
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<td>Minor second</td>
<td>Db</td>
<td>C#</td>
<td>25:3^5</td>
<td>1.053</td>
<td>90.2</td>
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<tr>
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<td>D</td>
<td>32:2^3</td>
<td>1.125</td>
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<td>Minor third</td>
<td>Eb</td>
<td>D#</td>
<td>35:3^3</td>
<td>1.186</td>
<td>294.1</td>
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<tr>
<td>Major third</td>
<td>MI</td>
<td>E</td>
<td>34:2^6</td>
<td>1.265</td>
<td>407.8</td>
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<tr>
<td>Fourth</td>
<td>FA</td>
<td>F</td>
<td>2^2:3</td>
<td>1.333</td>
<td>498.1</td>
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<td>GB</td>
<td>Gb</td>
<td>210:3^6</td>
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<td>588.3</td>
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<td>Fifth</td>
<td>SO</td>
<td>G</td>
<td>3:2</td>
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<td>Minor sixth</td>
<td>Ab</td>
<td>A#</td>
<td>27:3^4</td>
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<td>A</td>
<td>33:2^6</td>
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<td>Bb</td>
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<td>1.788</td>
<td>996.1</td>
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<tr>
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<td>Ti</td>
<td>B</td>
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<td>1.900</td>
<td>1109.8</td>
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<tr>
<td>Octave</td>
<td>DO</td>
<td>C</td>
<td>2:1</td>
<td>2.000</td>
<td>1200.0</td>
</tr>
</tbody>
</table>

Source: Burns and Ward 1982 and Martin 1962 by permission of publisher.

Courtesy of MIT Press. Used with permission.

Note F0 frequencies

Equally-tempered scale

The subscripts indicate octave (register). Note that the octave boundaries are at C, not A

Table removed due to copyright restrictions. Frequencies of notes in tempered scale.
Text description of just intonation and musical examples removed due to copyright restrictions.
Just intonation -- conclusions

- Although differences between just and equally-tempered tunings might not be discriminable when notes are presented sequentially (as for measuring JNDs),
- these differences can become apparent when notes are sounded together.

- The differences are somewhat subtle, but are most obvious when the notes are sustained harmonic complexes, e.g. organ-like.
Microtonal music

• http://infohost.nmt.edu/~jstarret/microtone.html

• "Most of the music we hear in based on a system called 12 tone equal temperament (or 12TET for short), where the octave is divided into 12 equal parts. Microtonal music is generally defined as any music that is not 12TET. Some folk musics are based on the harmonic series, some divide the octave into 19 or 31 equal parts, some divide the octave into 43 unequal parts, some don't divide the octave at all.... There are an infinite number of ways to choose your tonal resources."

see also http://www.corporeal.com/cm_main.html
1. Relations between tuning systems & dissonance:

For inharmonic instruments, there are situations where just tunings sound more dissonant. (His explanation for Gamelan tunings)

2. Experiments with alternate scales and tunings -- 10 TET music.

"Ten Fingers: If God had intended us to play in ten tones per octave, he would have given us ten fingers."
Harry Partch

- Harmonic Canons: there are two types. The simpler type is Harmonic Canon II; these box-like instruments have 44 strings and adjustable bridges which are uniquely configured for each piece. Harmonic Canon I has two planes of 44 strings each. The planes intersect near the middle of each string and thus the player may play on either plane or both at once. Also a moveable pyrex rod controls the pitch on some strings in one plane. The harmonic canons are both melody instruments and as providers of the harmonic underpinning, hence its name, canon, used in the sense of "law". It is played with picks or fingers and is strikingly used in cascades of pitches. Partch built his first canon in 1945, and continued to refine the instrument into the 70s.
History of scales

Pentatonic (5-note), heptatonic scales

Diatonic scales (Western tradition, 7 notes, “heptatonic”)

Ecclesiastical modes (traditional & medieval church music)

Common practice period (1600-1900) streamlined scale systems to two modes, Ionian (major scale) and Aeolian (minor scale), for reasons that may have to do with the difficulties of harmonic construction (e.g. in modulating between keys).

We will discuss major and minor scales first, then modal scales.
Major and minor scales

- **Major scale** pattern:
  - T-T-S-T-T-T-S

- **Minor scale** pattern:
  - T-S-T-T-S-T-T

Figure 10.3
Major and minor scales. Two octaves on the piano keyboard are sketched at the top. Each black key has two possible note names: the sharp above the lower white key and the flat below the upper white key (e.g., C$\#$/D$\flat$, D$\#$/E$\flat$, etc.). Major scales beginning on the first three white keys are illustrated in (a). The scale note and the interval size are indicated. The minor scale beginning on the first white key is illustrated in (b). The equivalence of one major and one minor scale is illustrated in (c). The sixth note of the major scale is the first (or tonic) note of the minor scale.

Courtesy of MIT Press. Used with permission.
Key relations of diatonic scales in terms of shared notes

Major scales & minor scales sharing same notes

Each step around the circle is a change of 1 note in common.

Distance around the circle is a measure of key distance

Note relatedness, distance from each other from the tonic.

Figure by MIT OpenCourseWare.
Modes show the effects of different scale types

Dorian

Scarborough Fair/Canticle
Simon & Garfunkel

http://www.8notes.com/articles/modes/

Musical example removed due to copyright restrictions.

Gaillarde I-II-II
Convivum Musicum Gothenburg

Thursday, May 14, 2009
Modes

Major scale

- Ionian mode
- Dorian
- Phrygian
- Lydian
- Mixolydian
- Aeolian

Minor scale
Why do they sound different, despite the same note set? Because the notes have different patterns of relations to the tonic, and particular musical intervals may be present or absent from a given modal scale.

“Happy Birthday” in different modes

How would we distinguish C-major from A-minor key (or any of the other modes below, for that matter) if they have the same note set?

**Establishment of the tonic, and its consonant intervals.**

**Major scale** pattern:
- T-T-S-T-T-T-T-S

4th & 5th
No tritone

**Minor scale** pattern:
- T-S-T-T-S-T-T

No 4th
Tritone

Thursday, May 14, 2009
Lydian mode example

The Lydian Liver

Church Lydian Mode

Of all the organs deep inside of me, I know my liver best. It has an edge which anyone can feel, 'cause it's bigger than the rest. It makes my albumin and my clotting factor one, and it fills my gut with bile. And removes ammonia from my blood, which always makes me smile.

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http://www.pathguy.com/modes.htm#phrypara

Thursday, May 14, 2009
Phrygian mode

Phrygian Paraquat Dirge

Church Phrygian Mode

E minor   G7    E minor

I'm sorry I drank paraquat. It's

E minor   G7    E minor   G7    G7    A minor

diffuse air space damage I've got. Now my lungs are all filled with the

A minor   G7    G7

cells that I killed, and with fibrin, scar tissue, and snot.

Courtesy of Ed Friedlander.

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http://www.pathguy.com/modes.htm#phrypara
An "exotic" mode

"The Answer is Dark" is an example of one of those more exotic modes. It's in a mode of a scale that uses 3 half-step intervals, causing one interval to be an augmented second. In the chart below, the bottom line designates the type of interval between each member of the scale where: h = half step a = augmented whole step w = whole step

G#  A  B#  C#  D#  E#  F#  G#

      h  a  h  w  w  h  w

i.e. S,T+,S,T,T,S,T

Listen to a MIDI file of the scale mode:
It would be in G# Mixolydian except that the A# has been lowered to A. This is sometimes called a "mixed mode" because it has characteristics of two diatonic modes.

http://www.elvenminstrel.com/tolkien/modes.htm
According to the nomenclature of medieval music theorists, who were dealing largely with unchorded plainsong, our natural major is the church "Ionian Mode" (C-D-E-F-G-A-B-C), and our natural minor is the church "Aeolian mode" (C-D-D#-F-G-G#-A#-C). I became curious about modes when I learned that "Wreck of the Edmund Fitzgerald" and "Scarborough Fair" use the old balladic scale which matches the church "Dorian Mode" (C-D-D#-F-G-A-A#-C). I used the eerie church "Mixolydian mode" (C-D-E-F-G-A-A#-C) for my intranet version of "The Pathology Blues", on our quizbank. You can also hear the church "Mixolydian mode" in "The Beat Goes On", "Luck Be a Lady Tonight", "Norwegian Wood", "Day Tripper", "Sundown" (Gordon Lightfoot), "Cats in the Cradle", "City of New Orleans" (verse but not chorus), and the theme to "Star Wars". Caedmon recordings used it for the tune for the mystical first song in Yeats's play "The Only Jealousy of Emer". The other church modes are novelties at best. Some of the old Gregorian chant "Sing my tongue..." seems to be Phrygian mode. There is some of the church "Phrygian mode" in "Fiddler on the Roof", and if the song is fully transposed into the church Phrygian mode, it still sounds okay. My own attempt to write a song using the church Phrygian mode was dismal. I wrote a little song in an unabashed church "Lydian mode". The mode itself suggested the subject. Unless you only use the subdominant as a leading tone for the dominant, any melody you write in this "mode" will be unnerving -- the subdominant is equidistant from the lower and upper tonics. A correspondent pointed out that the "Lydian" mode makes up some of the "Jetsons" and "Simpsons" theme. Bartok wrote a short piece in the Lydian mode. In the Locrian mode, the dead-center position of the dominant makes this even more unmusical. A music professional told me once that no ethnomusicologist has ever documented a folk tune in what medieval theorists called the "Locrian mode". I browsed a little in Plato, Aristotle, pseudo-Plutarch's "De Musica", and of course the Oxford History of Music, and came away wondering if the medieval music theorists (Boethius, Gregory the Great, their successors) really meant the same thing as did the Greeks who named the modes. Today most people (following a scholar named Westphal) tell us that the Greek modes were indeed used as "scales" with the tonic notes being the low-pitched one, just as the church mode theorists say. This seems to be based on statements in Plato and Aristotle that the modes had distinct emotive qualities, as our major and minor scales do. Another school of thought (that of Munro) claims that for the ancients, the modes were actually keys, i.e., you could play any melody in any mode. If this is true, then the ancient Greeks had either perfect pitch or a standard pitchpipe. I think people have probably liked similar tunes in different eras. I tried to figure out how the ancient Greeks would have played some of our favorites. Ancient Greek lyres typically had seven strings. (Some Hebrew lyres must have had ten strings -- see Psalm 33.) The system of modes is also called "harmoniae", which meant "fitting" or "tuning". Greek writers on music talk about the normal tuning comprising two tetrachords, i.e., a series of four notes with the lowest and highest separated by a major fourth and sharing the center string. Pythagoras and Terpander are both credited with the idea of having the highest string be an octave of the lowest string.

http://www.pathguy.com/modes.htm

Courtesy of Ed Friedlander.
 Scala is a freeware software application for experimenting with musical tunings. See http://www.huygens-fokker.org/scala/
What are Modes of a Scale?

"A modal melody is one that is not in a "key" of the major or minor sort common in music of the Western world in the past 300 to 400 years. Instead, its scale may be comprised of almost any set of tones, not only those that imply 3-part harmony. In fact, generally it should not imply 3-part harmony and a bass line. Many of the modal melodies I like best are virtually impervious to triadic harmonization. There are two common types of modal scales: pentatonic and heptatonic, meaning 5-tone and 7-tone, respectively. Folk music from many parts of the world, including Celtic, American, African, and Chinese, often use modes of pentatonic scales."

Listen to a MIDI file of two pentatonic modes:

http://www.elvenminstrel.com/tolkien/modes.htm
Important points

- Scales are anchored pitch systems
- The anchor is the tonic, first note in the scale
- The pitch intervals of the notes re: the tonic give the scale its characteristic tonality
- Scales include both consonant and dissonant intervals that provide tonal contrast.
- Equal temperament tuning systems use logarithmic frequency spacings to achieve systematicity (equality of keys good for key modulations).
- The 12-TET system represents just frequency ratios (2:1, 3:2, 4:3, 5:4, 5:3) fairly well -- important because of consonance, fusion, pitch stability.
"As Frank Zappa explained it, "The creation and destruction of harmonic and 'statistical' tensions is essential to the maintenance of compositional drama. Any composition (or improvisation) which remains consistent and 'regular' throughout is, for me, equivalent to watching a movie with only 'good guys' in it, or eating cottage cheese."[2] In other words, a composer cannot ensure a listener's liking by using exclusively consonant sounds. However, an excess of tension may disturb the listener. The balance between the two is essential." (Wikipedia)
Reading/assignment for next meeting

Book reviews due TODAY
Harmonics & Scales Problem set due next MONDAY.

• We will do Harmony & Melody next time.
• Reading:
  • You may find Wikipedia entries on consonance and dissonance, melody and harmony useful.
  • Do read Chapters 8 & 9 in Aiello on melody (Butler & Brown) and tonal expectancies (Bharucha).
  • Handel, Chapter 10, "Grammars of music and language" on Blackboard also covers similar territory.

Thursday, May 14, 2009