MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Departments of Electrical Engineering, Mechanical Engineering, and the Harvard-MIT Division of Health Sciences and Technology

6.022J/2.792J/BEH.371J/HST542J: Quantitative Physiology: Organ Transport Systems

QUIZ 1

SOLUTIONS

Problem 1

A. Draw normal $P(t)$ waveforms for the left ventricle, left atrium, and aorta. Show two complete cardiac cycles, and use typical normal values for the pressures. Use the time axis provided in Figure 1.1a, and assume a heart rate of 60 bpm.

$$C.O. = \frac{\dot{V}_{O_2}}{C^A_{O_2} - C^V_{O_2}} = \frac{363 \ ml\ O_2/min}{(200 - 14.5)\ ml\ O_2/liter\ blood} = \frac{363}{55} = 6.6\ L/min$$

So

$$SV = \frac{6.6\ L/min}{60\ beat/min} = 110\ cc/beat$$

(See Figure.)

B. The cardiac output was measured using the Fick method.

Oxygen uptake	363 ml O_2 per minute
Arterial oxygen content	200 ml O_2 per liter of blood
Mixed venous oxygen content	145 ml O_2 per liter of blood

Using this data together with your $P(t)$ waveforms, draw the corresponding P-V loop for the LV. Assume an end-diastolic LV volume of 170 cc., and a LV "dead" volume of 15 cc for both systole and diastole. Draw linear systolic and diastolic P-V curves, and use the axes provided.

See Figure.

C. Correlate the following landmarks on the P-V loop with the appropriate points on the $P(t)$ curves using the numeric labels below:

a: begin LV contraction

b: peak LV pressure

c: begin LV filling

d: end ejection

e: begin LV ejection

See Figure.

D. "Ejection fraction" (EF) is defined as the percentage of the end-diastolic volume that is ejected during systole. What is the EF in this case? (Normal > 55%.)

$$EF = \frac{110}{170} = 64.7\%$$

E. A papillary muscle in the LV ruptures. (Assume that there are no functioning controls, and that the system has reached a new steady state.) The new arterial BP (systolic, diastolic, and mean) drops to 60% of its original value.

New BP:

$$\begin{aligned} 120 \times 0.6 &= 72 \\ 80 \times 0.6 &= 48 \\ 110 \times 0.6 &= 66 \end{aligned}$$

(i) Sketch two cardiac cycles showing the new $P(t)$ waveforms, using the axes supplied in Figure 1.1b. Pay particular attention to the new amplitudes of the LV and LA pressures. Assume no change in the left ventricular end-diastolic pressure and volume.

See sketches for $P(t)$, P-V loop. Note high pressure in atrium at end-systole due to blood leaking past mitral valve.

(ii) Sketch the new P-V loop on the same axes as part (B) above. Estimate the new stroke volume.

New stroke volume $\approx 170 - 42 = \boxed{128\ cc}$

(iii) What is the new ejection fraction (using the definition in part D)?

$$EF' = \frac{128}{170} = 75.3\%$$

(iv) Crudely approximate the stroke volume delivered to the aorta by making use of the Windkessel approximation.

The pulse pressure had decreased from

$$120 - 80 = 40\ mmHg$$

to

$$72 - 48 = 24\ mmHg\ \textit{(60\% of prior)}$$

So, if we assume SV is proportional to pulse pressure,

$$SV' = 0.6 \times \underset{\substack{\uparrow \\ \textit{original SV}}}{110}\ cc = \boxed{66\ cc}$$

(v) What is the “forward ejection fraction” (the percentage of the end-diastolic LV volume that is ejected into the aorta)?

$$\textit{Forward EF} = \frac{66}{170} = 38.8\%$$

(vi) As a result of the papillary muscle rupture, a murmur appears. Indicate its temporal location on the time axis provided in Figure 1.1.

It is heard throughout systole as a regurgitant jet enters the atrium. See Figure.

Figure 1.1:

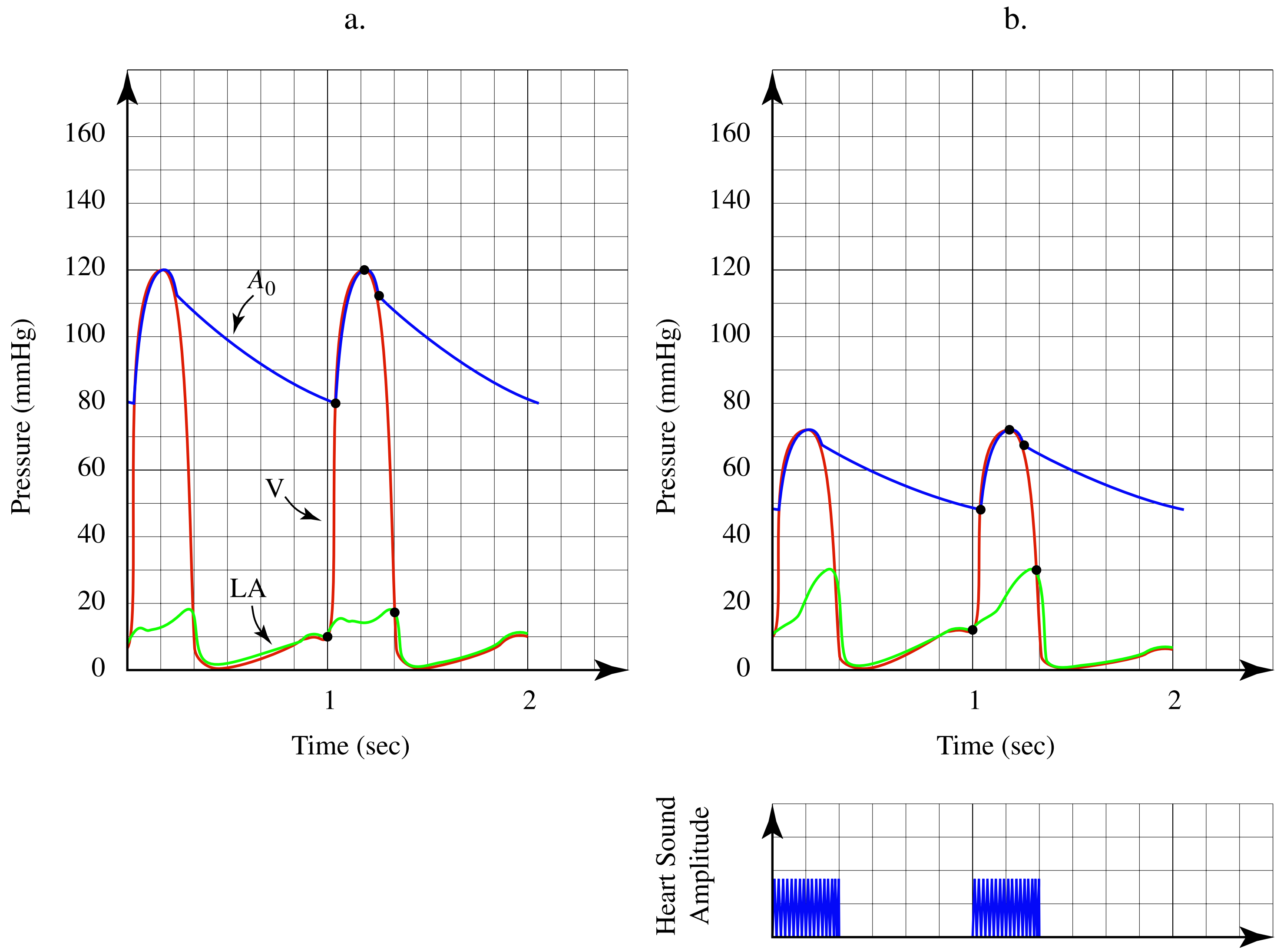
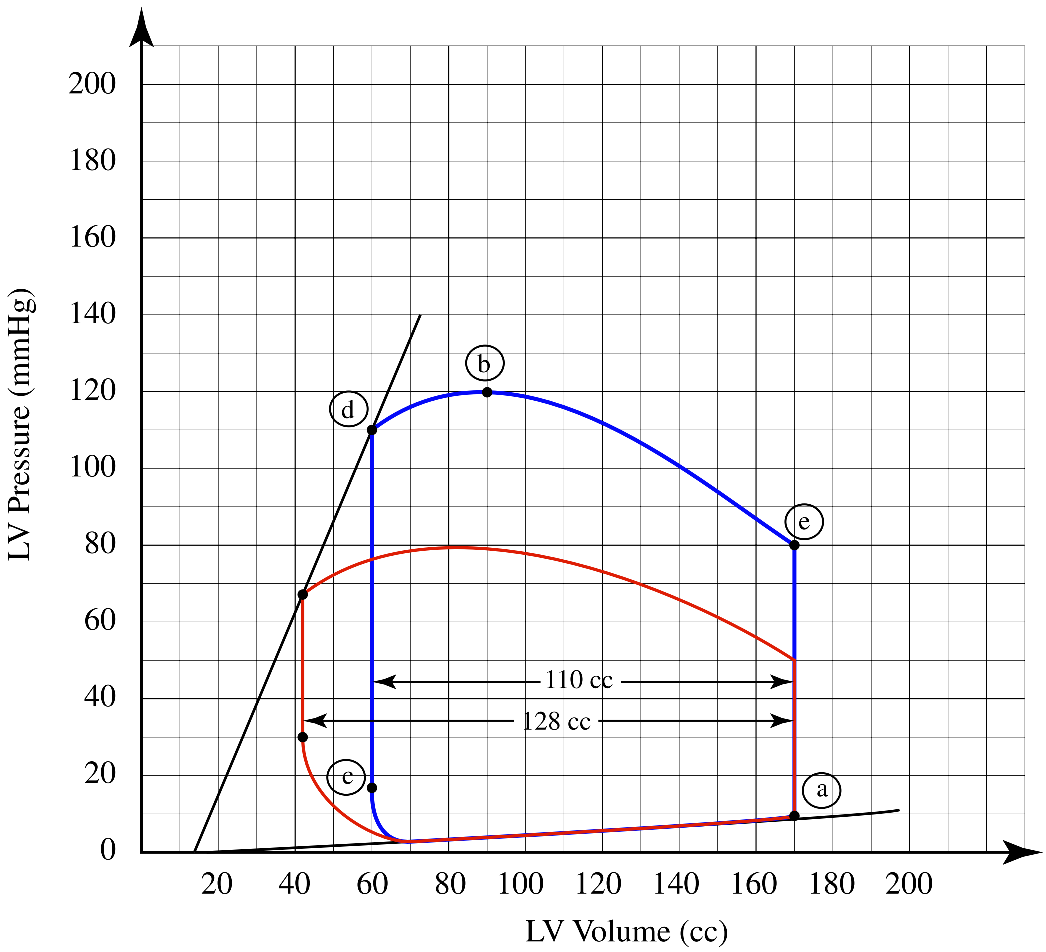


Figure 1.2:



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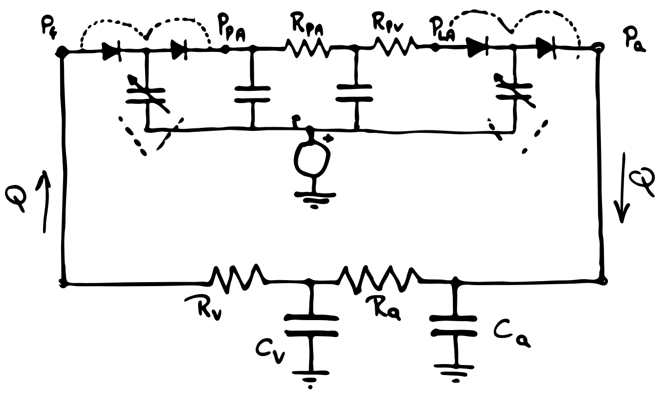
Problem 2

We have used the lumped parameter model of the cardiovascular system that is shown in Figure 2.1. The following relationship was derived to relate cardiac output to the various model parameters (in operating region I):

$$\text{C.O.} = \frac{P_{ms} - P_{th} - \left(P_{PA}^{0} - P_{th}\right)\frac{C_S^r}{C_D^r}}{R_v + R_a\frac{C_a}{C_a+C_v} + \frac{1}{fC_D^r}}$$

Figure 2.1: Lumped Parameter Model



Part 1

Using this expression and/or graphical analysis explain the expected changes in: (a) *cardiac output*, (b) *arterial blood pressure*, and (c) *pulse pressure* that would result from the following interventions, assuming an uncontrolled CV system and a heart rate of 60 bpm.

A. Increasing the peripheral resistance, R_a.

B. Decreasing total blood volume.

C. Increasing left ventricular contractility.

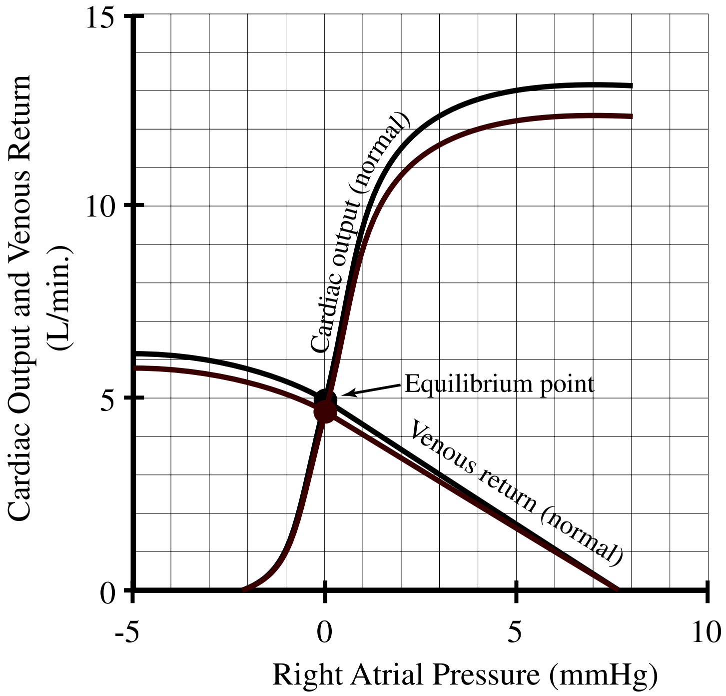
D. Decreasing arterial capacitance, C_a, by a factor of two.

E. Increasing the intra-thoracic pressure by 10 mmHg, and P_{ms} by 8 mmHg by blowing into a balloon.

Part 2

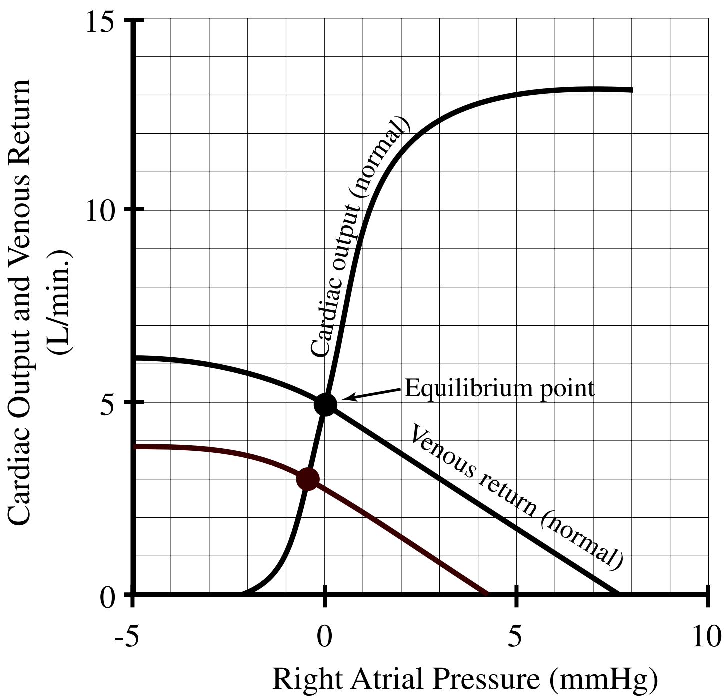
For each intervention above, sketch the expected qualitative changes in the CO/VR curves using the graphs below.

A. Increasing the peripheral resistance, R_a.



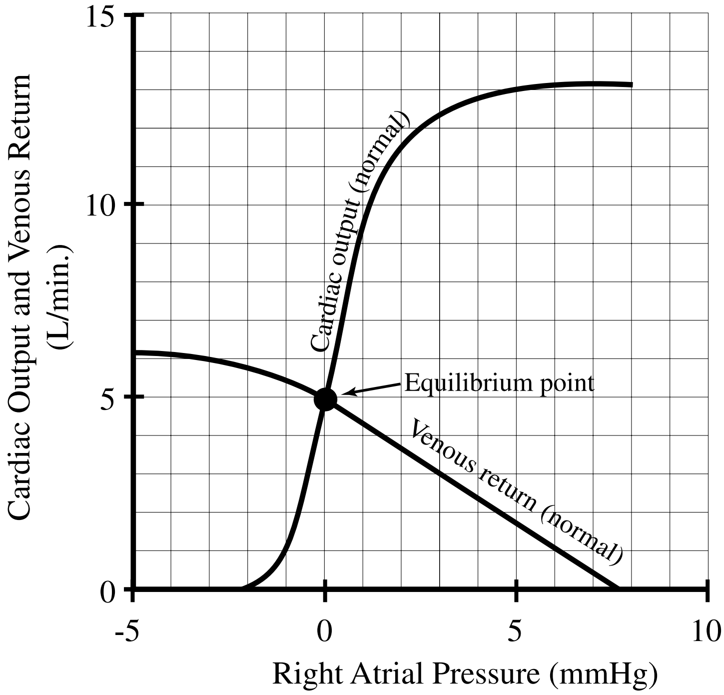
$\overline{ABP} = CO \times R_a$. *There will be a slight decrease in CO, a large increase in* $\overline{ABP}$*, and a change in the* τ *of ABP decay, but little change (a slight decrease) in pulse pressure.*

B. Decreasing total blood volume.



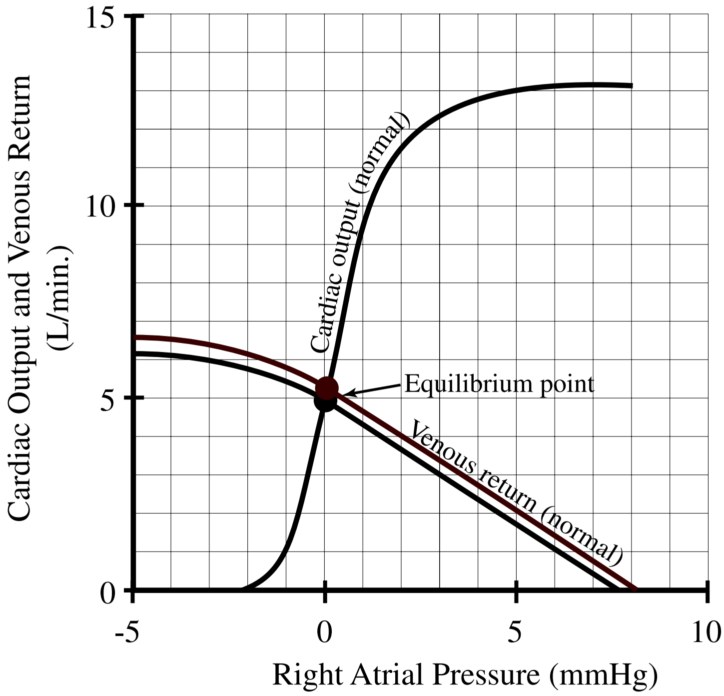
P_{ms} drops and the VR curve shifts to the left. CO drops. ABP drops proportionately because $\overline{ABP} = C.O. \times R$. Since HR does not change, pulse pressure also drops proportionately.

C. Increasing left ventricular contractility.



CO is not a function of C_S^L*, so there is no change in CO,* $\overline{ABP}$*, or PP.*

D. Decreasing arterial capacitance, C_a, by a factor of two.



There is a slight increase in CO from 5.44 *L/min to* 5.96 *L/min.*

$C_a = 2:$

$$\begin{aligned} P_{ms} &= \frac{V_T - V_0}{C_a - C_v} = \frac{4000 - 3200}{2 + 100} = \frac{800}{102} = 7.84 \\ CO &= \frac{7.84 - (-5) - [15 - (-5)]\frac{2}{20}}{.05 + 1 \cdot \left(\frac{2}{102}\right) + \frac{1}{1 \times 20}} = \frac{7.84 + 5 - 2}{.05 + .0196 + .05} = \frac{10.84}{.1196} = 90.63 \ cc/sec \\ &= 5.44 \ L/min \end{aligned}$$

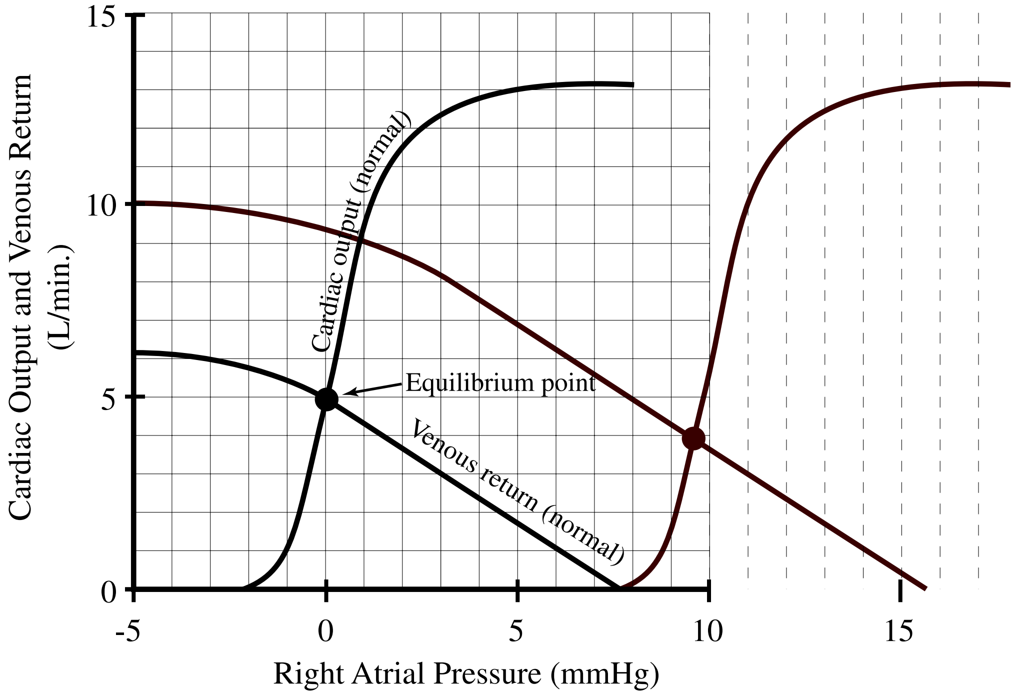
$C_a = 1:$

$$\begin{aligned} P_{ms} &= \frac{800}{101} = 7.92 \\ CO &= \frac{7.92 + 5 - 2}{.05 + 1 \cdot \left(\frac{1}{101}\right) + .05} = \frac{10.92}{0.1099} = 99.36 \ cc/sec = 5.96 \ L/min \end{aligned}$$

Mean ABP will rise proportionately. The pulse pressure, however, will double *since*

$$Pulse\ Pressure \approx SV/C_a$$

E. Increasing the intra-thoracic pressure by 10 mmHg, and P_{ms} by 8 mmHg by blowing into a balloon.



Original CO by equation:

$$CO \approx \frac{(7.8+5)-(15+5)\frac{2}{20}}{.05+.02+.05} = \frac{12.8-2}{.12} = \boxed{5.4\ L/min}$$

With new P_{th}, P_{ms}:

$$CO' = \frac{(15.8-5)-(15-5)0.1}{.12} = \frac{10.8-1}{.12} = 81.67\ cc/sec = \boxed{4.90\ L/min}$$

Table 1: Glossary of Symbols and Nominal Value for Model Parameters

Symbol	**Definition**	**Normal Value**
ΔV	stroke volume	96 cc
$f = \frac{1}{T}$	heart rate	60/min. = 1/sec.
$T = T_S + T_D$	duration of heart cycle	1 sec.
T_S	duration of systole	.3 sec.
T_D	duration of diastole	.7 sec.
C_D^r	diastolic capacitance of RV	20 ml/mmHg
C_D^l	diastolic capacitance of LV	10 ml/mmHg
C_S^r	minimum systolic capacitance of RV	2 ml/mmHg
C_S^l	minimum systolic capacitance of LV	.4 ml/mmHg
V_{max}^r, V_{max}^l	"maximum" volumes, RV, LV	200 cc
$V_T = V + V_0$	total volume of blood in peripheral vasculature	4000 ml
V_0	volume needed to fill peripheral vasculature without increasing pressure	3200 ml
C_a	arterial capacitance	2 ml/mmHg
C_v	venous capacitance	100 ml/mmHg
R_a	arterial resistance	1 mlHg/(ml/sec)
R_v	resistance to venous return	.05 mmHg/(ml/sec)
P_{th}	mean intrathoracic pressure	-5 mmHg
P_A^0	pulmonary artery pressure (end-systolic) referenced to mean intrathoracic pressure	15 mmHg
P_{ms}	mean systemic filling pressure (see text)	7.8 mmHg
P_v	peripheral venous pressure	6.1 mmHg

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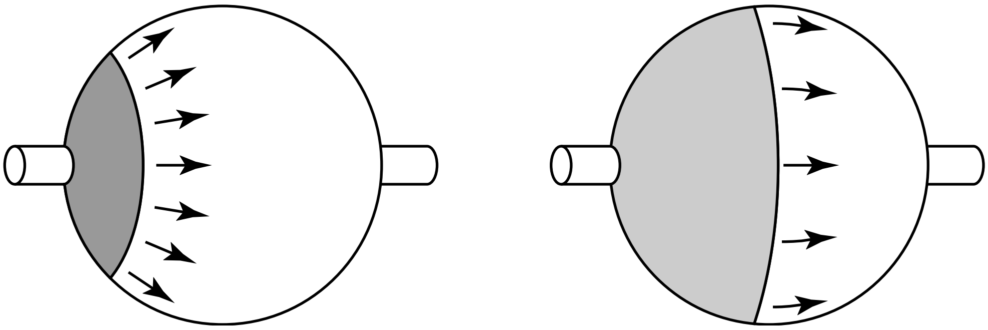
Figure 3.3:

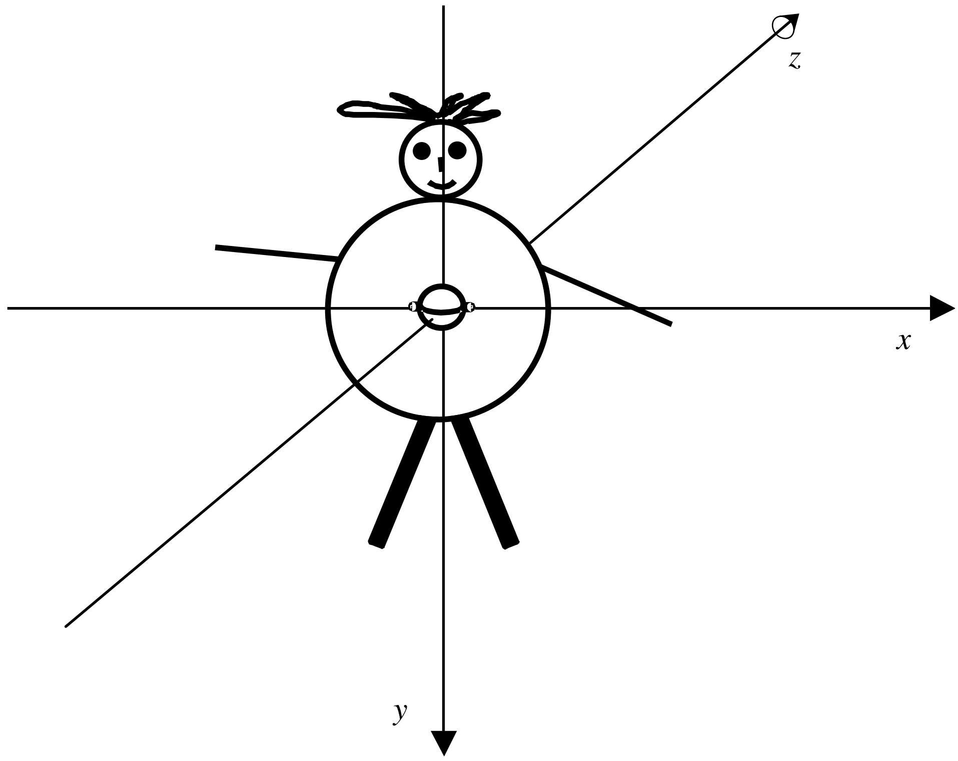


sphere is depolarized before repolarization begins.

Assume the heart to be in the center of the spherical torso, and that all the assumptions underlying the dipole ECG theory are valid.

A. Sketch the three orthogonal scalar waveforms $V_x(t)$, $V_y(t)$, and $V_z(t)$ as defined in Figure 3.4 for one depolarization sequence. Label the time axis in terms of the radius of the spherical heart, a, and the velocity of propagation, v. [Note: try to be as quantitative as possible, but partial credit will be given for a qualitative answer.]

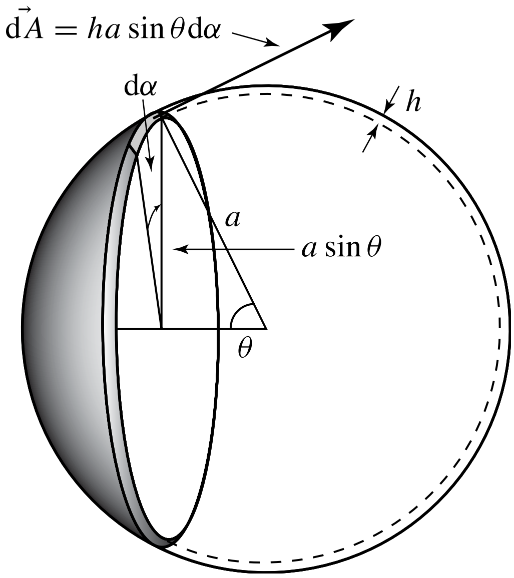
Figure 3.4:



Symmetry leads to cancellation of the y and z components of the heart vector. So only V_x is non-zero.

First, calculate the equivalent heart vector, $\vec{M}_0$. We know it is in the x-direction. Its magnitude is the projection on $\vec{i}_x$ of the individual components.

Figure 3.5:



Let h be the thickness of the shell.

At the interface of depolarized and polarized tissue there is a circular boundary of radius $a \sin\theta$. An elemental area, $\mathrm{d}A$, may be defined as

$$\mathrm{d}A = ha \sin\theta \mathrm{d}\alpha$$

where α is the angle of rotation around the x-axis. Let $\vec{m}$ be the elemental current dipole per unit area, and the dipole moment associated with $\mathrm{d}A$ will be

$$\vec{m} = mha \sin\theta \mathrm{d}\alpha$$

$\vec{m}$ makes an angle $\left(\frac{\pi}{2} - \theta\right)$ with the x-axis for all α. The projection of $\vec{m}$ on $\vec{i}_x$ will therefore be

$$m_x = mha \sin^2\theta \mathrm{d}\alpha$$

The total net x-projection at a given θ would be

$$M_x(t) = \int_0^{2\pi} mha \sin^2\theta(t) \mathrm{d}\alpha = 2\pi mha \sin^2\theta$$

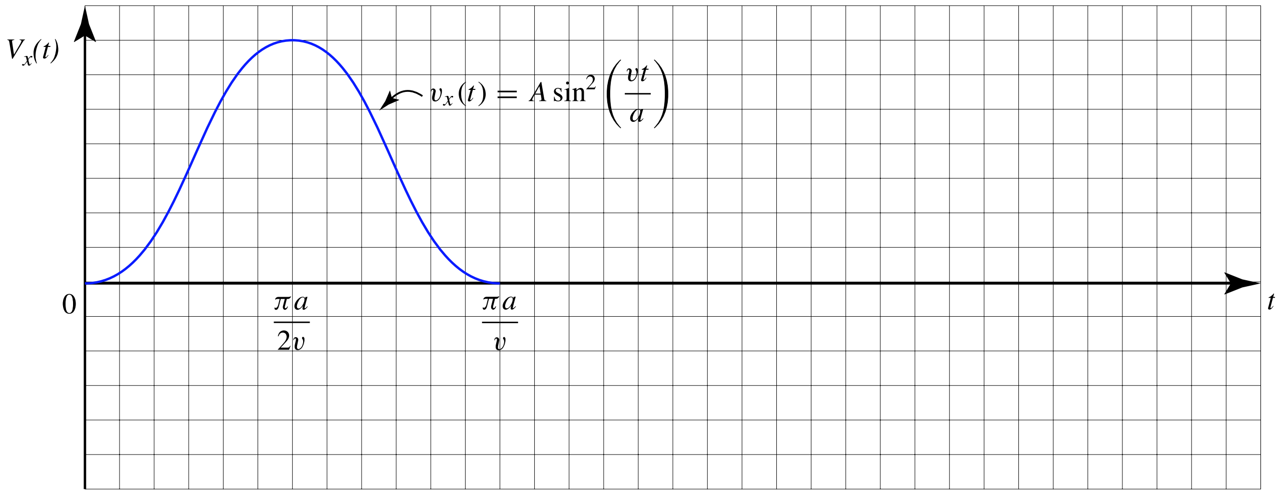
But

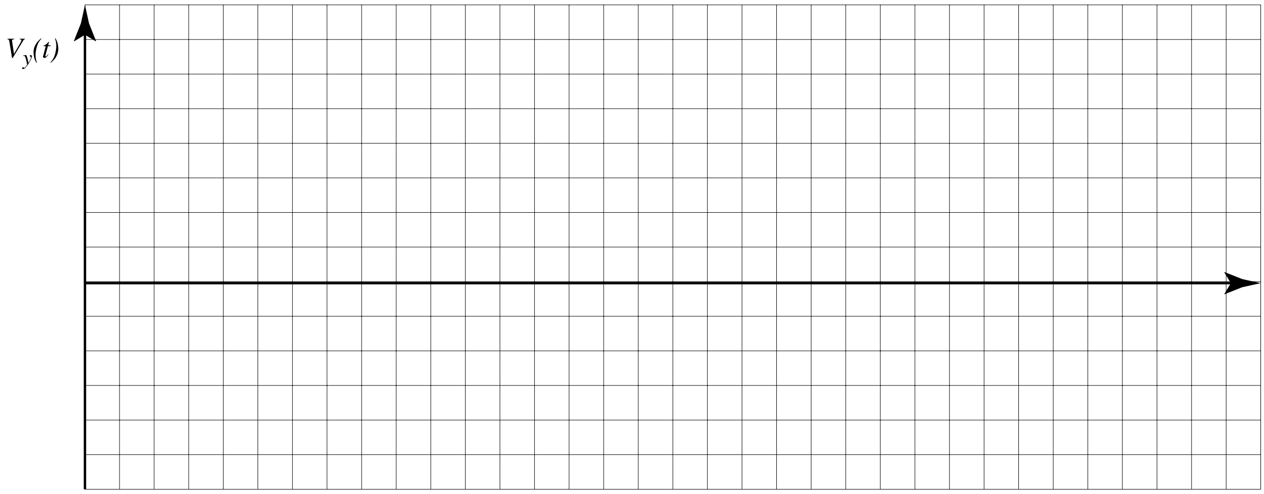
$$\theta = \omega t = \frac{v}{a}t$$

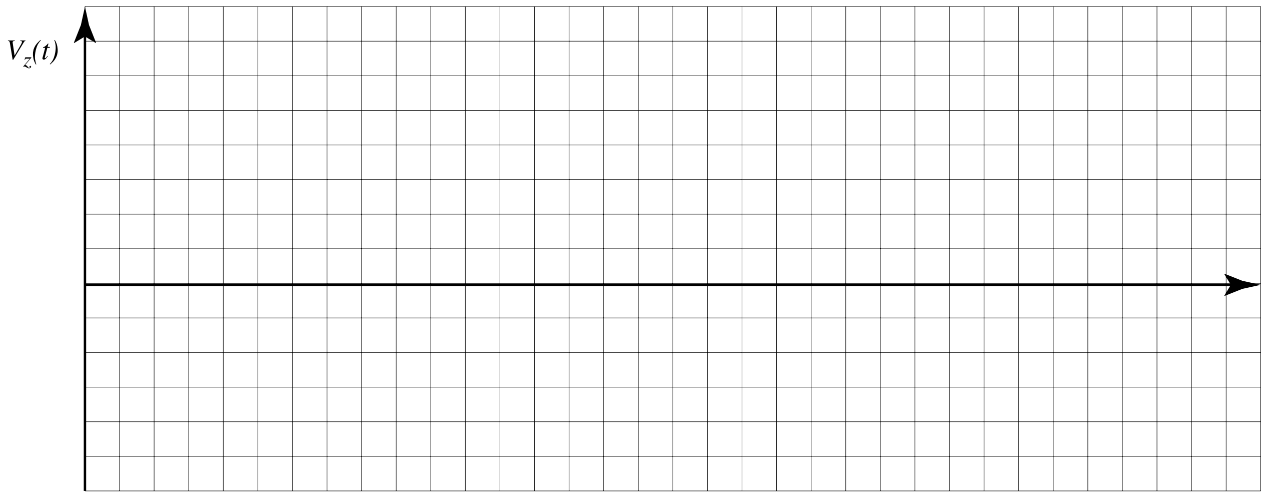
So

$$\boxed{M_x(t) = 2\pi mha \sin^2\left(\frac{vt}{a}\right)}$$

It is plotted in the figure below.


$V_x(t)$
$v_x(t) = A \sin^2\left(\frac{vt}{a}\right)$
0
$\frac{\pi a}{2v}$
$\frac{\pi a}{v}$
t


$V_y(t)$


$V_z(t)$