HST.582J / 6.555J / 16.456J Biomedical Signal and Image Processing
Spring 2007

HST-582J/6.555J/16.456J-Biomedical Signal and Image Processing-Spring 2007

Problem Set 1

DUE: 2/15/07

In this and other problem sets, you are encouraged to use *Matlab* (available on MIT Server) in order to evaluate numerical expressions and display graphs of mathematical functions. You may also wish use symbolic mathematical software, such as *Maple*, which is also available on MIT Server.

Problem 1

If you have ever seen a Western, you must have noticed that, as the stagecoach changes speed, the wheel appears to reverse direction even though the coach moves forward. This is an example of *aliasing*. The sampling mechanism is the movie camera capturing images at a frequency of 24 frames/second. The eye acts as a lowpass filter, in the sense that successive images reaching the eye at a rate exceeding 10 images/second appear as if they are superimposed. This is called the *critical fusion frequency*.

Answer the following questions assuming that the wagon wheel is 1.6 m in diameter and has 12 spokes.

(a) Express the true angular velocity of the wheel (in radians/second) as a function of the coach velocity.

(b) At what set of coach velocities does the wheel appear to reverse direction?

(c) Determine an expression for the *apparent* angular velocity of the wheel as a function of the coach velocity.

(d) How many spokes does the wheel appear to have the first time it appears to reverse direction? The second time? Explain the difference between these two cases.

Problem 2

(For this problem, you may find it helpful to refer to Section 1.A.1 in the notes.) Consider the continuous-time periodic signal, $x(t)$, with period T:

$$x(t) = t \qquad\qquad 0 < t < T.$$

(a) Find the Fourier series representation of $x(t)$. Express it in both sine-cosine form and exponential form.

(b) What is the physical significance/function of the Fourier series coefficient corresponding to $k = 0$ (that is, A_0 or X_0)?

(c) For $T = 5$, plot the approximation to $x(t)$ obtained from the first $2N + 1$ Fourier series coefficients, for the three cases corresponding to $N = 1$, 3, and 10. (For the sine-cosine form, the first $2N+1$ coefficients correspond to $k = 0, \ldots, N$; for the exponential form the first $2N+1$ coefficients correspond to $k = -N, \ldots, N$.)

(d) What is the total energy in one period of $x(t)$? What percentage of the total energy is accounted for by the first $2N+1$ Fourier series coefficients as a function of N? Evaluate this expression for $N = 1, \ldots, 10$? (HINT: Use Parseval's theorem.)

We wish to use a conventional A/D converter to convert the continuous-time signal, $x(t)$, to a quantized, discrete-time signal, $x[n]$.

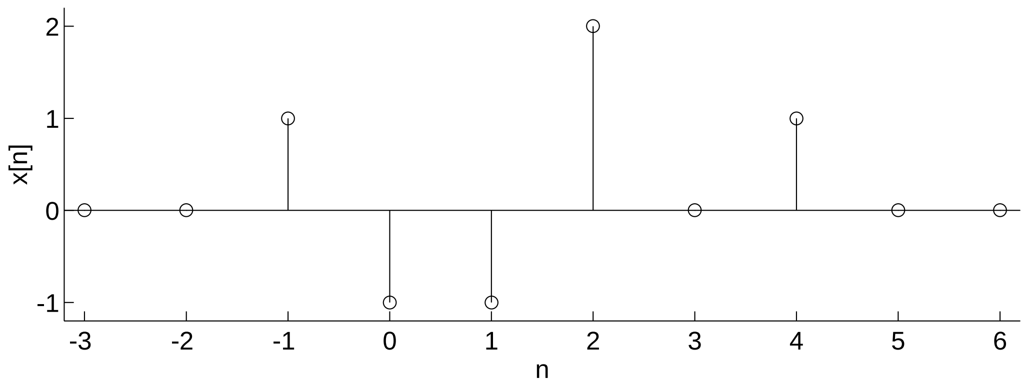
(e) Based on the results of part **(d)**, what sampling rate is required to insure that less than 2% of the total signal energy is aliased?

(f) What alternative is available if we require that virtually no signal energy is aliased to lower frequencies?

(g) How many bits must the quantizer have in order to insure that the quantization noise will be 60 dB below the signal power? Be sure to state your choice of $V_{\max}$.

Problem 3

Consider the signal $x[n]$ shown below. $x[n] = 0$ for $n \leq -2$ and $n \geq 5$.



(a) Sketch $x[-n]$, $x[n+2]$, $x[n-2]$, $x[-n+2]$, and $x[-n-2]$.

Consider the digital filter described by

$$y[n] = \sum_{m=-\frac{N}{2}}^{\frac{N}{2}} a^m x[n-m].$$

(b) Find $h[n]$, the impulse response of this filter. Is it an FIR filter or an IIR filter? Is it stable? Is it causal?

(c) Sketch $h[n]$ for $a = 0.5$ and $N = 4$. Determine $y[n]$ for these values using $x[n]$ given in the figure above.