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HST.582J / 6.555J / 16.456J Biomedical Signal and Image Processing
Spring 2007

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HST-582J/6.555J/16.456J-Biomedical Signal and Image Processing-Spring 2007

Problem Set X

- QUIZ 1 will take place on Tuesday, March 20, from 9:30-11 am in 56-154 (usual time and place).
- The quiz will cover material presented in lecture through Thursday, March 8.
- The quiz will be closed book. One $8\frac{1}{2} \times 11$ inch sheet of notes (both sides) will be allowed.
- Coverage of topics on the quiz will be somewhat representative of the amount of time spent on each topic in lectures, labs and problem sets. You are *not* responsible for material in the course notes that was not covered elsewhere in the course. Please see the annotated Outline of Course Notes for listings of specific topics.
- This ungraded problem set will help you prepare for the quiz in two ways. First, it will give you a chance to think about solving problems in imaging processing, which was not covered in any previous problem set. Second, it illustrates the type of questions asked on previous years' quizzes.
- This problem set will not be collected or graded. Solutions will be posted on the course website by Thursday, March 15.

Problem 1

Consider the following small image for parts **a)** and **b)**:

```
1  2  3  4  5  6  7
1  2  3  4  5  6  7
1  2  3  4  5  6  7
1  2  3  94 5  6  7
1  2  3  4  5  6  7
1  2  3  4  5  6  7
1  2  3  4  5  6  7
```

a) Calculate the image that results from convolution with the following kernel:

```
1 1 1
1 1 1
1 1 1
```

Don't worry about boundary values – just provide the result for the central 5 by 5 part of the result.

b) Calculate the image that results from applying a 3×3 median filter to the image described above.

Again, don't worry about boundary values – just provide the result for the central 5 by 5 part of the result.

Problem 2

The *gradient* of a two-dimensional function is a vector-valued derivative. The x component of the gradient is defined by $\frac{\partial f(x,y)}{\partial x}$.

Consider the following three discrete approximations to the x component of the gradient of an image f . For each case, determine a convolution kernel that produces an image containing the given approximation of the x component of the gradient.

a) $f(n_1 + 1, n_2) - f(n_1, n_2)$

b) $f(n_1, n_2) - f(n_1 - 1, n_2)$

c) $.5 * [f(n_1 + 1, n_2) - f(n_1 - 1, n_2)]$

Problem 3

- a) Consider the two-dimensional “boxcar” function defined to have unit amplitude when $|t_1| < T$ and $|t_2| < T$ and zero otherwise. If this function is viewed as the impulse response of a filter, how would you characterize the filter: “high-pass”, “low-pass”, or “band-pass”?
- b) Next, examine the discrete analogue of the above function, which is defined to have unit value when $|n_1| < N$ and $|n_2| < N$. Now consider this function as a kernel for convolution with some discrete 2D signal. In a *brute-force* convolution by this kernel, approximately how many arithmetic operations are required *per-pixel of the result*?
- c) The above discrete kernel is an example of a “separable” kernel. When taking advantage of this separability in a convolution, approximately how many arithmetic operations are required *per-pixel of the result*?
- d) Next consider implementing the above convolution using FFT methods. Once again, approximately how many arithmetic operations are required *per-pixel of the result*, on a signal with size M by M ?
- e) Can you think of an implementation of the above 2D FIR filter, not using transforms, that computes the result with approximately four arithmetic operations *per-pixel of the result*? [Hint: exploit separability and use two passes of filtering, one with a “horizontal” kernel and one with a “vertical” kernel. Focussing on the filtering by the “horizontal” kernel, and assuming that you have just computed the result for some pixel, can you think of a cheap way to compute the result for the neighboring pixel on the right?]

The following problems are taken from last year’s quiz. Please note that the figures on pages 12 and 15 do not photocopy well, so you may wish to view them online.

Question 1 (50%) This question has eight parts, but please don't panic; many of them can be answered independently. Consider the continuous-time signal

$$x(t) = \begin{cases} \frac{\sin(20\pi t)}{20\pi t} & \text{for } -t_0 < t < t_0 \\ 0 & \text{otherwise} \end{cases}$$

Figure 1 shows three functions

$x(t)$ for $t_0 = 0.5$

$x[n]$, the discrete-time signal obtained by sampling $x(t)$ at $F_s = 100$ Hz

$X(f)$, the DTFT of $x[n]$

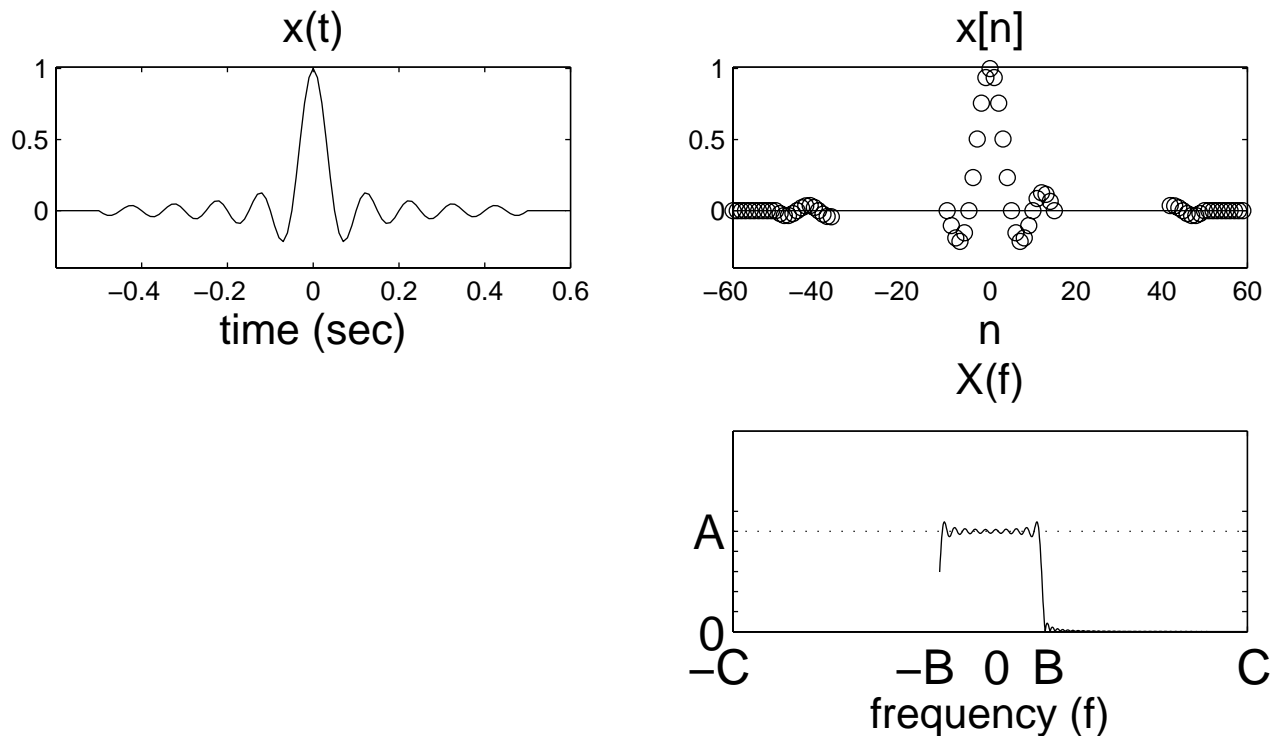


Figure 1:

(a) Determine the values of A, B, and C for $X(f)$, the DTFT of $x[n]$, in Figure 1.

A =

B =

C =

(b) Is there a value of t_0 that makes $X(f)$ a perfect rectangle? If so, state the value of t_0 .

YES / NO **circle one** If yes, $t_0 =$

(c) Is there a value of F_s that makes $X(f)$ a perfect rectangle? If so, state the value of F_s .

YES / NO **circle one** If yes, $F_s =$

Now consider a recently discovered alien species that has an ECG signal such that a single heartbeat equals $x(t)$ in Figure 1. A *normal* ECG reading for this alien, $y(t)$, consists of a sequence of these individual heartbeats defined by

$$y(t) = x(t) * p_1(t) \quad \text{where} \quad p_1(t) = \sum_{r=-\infty}^{\infty} \delta(t - r).$$

An *abnormal* ECG reading, $z(t)$, consists of a sequences of these individual heartbeats alternating with segments of zero signal, defined by

$$z(t) = x(t) * p_2(t) \quad \text{where} \quad p_2(t) = \sum_{r=-\infty}^{\infty} \delta(t - 2r).$$

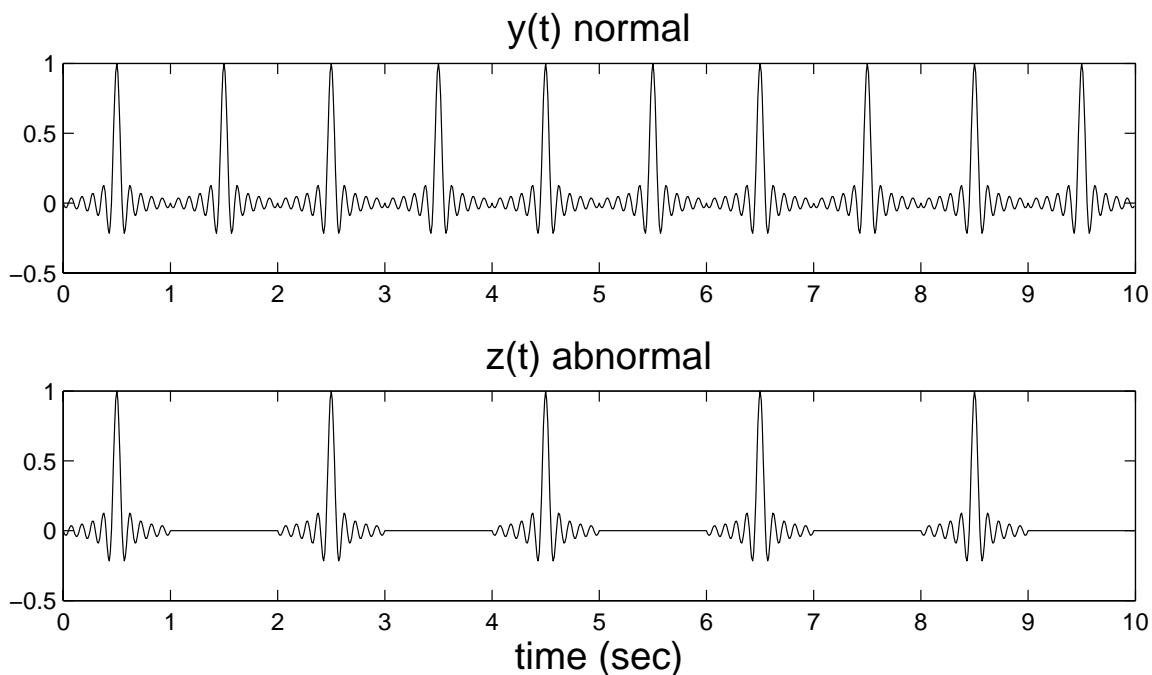


Figure 2:

(d) $z(t)$ is the result of convolving $x(t)$ with an impulse train with a period of $T = 2$ sec. Circle the appropriate choice on each line to make the following sentence true.

$Z(F)$, the CTFT of $z(t)$, will be the result of *adding* / *multiplying* / *convolving*

$X(F)$ with a periodic *boxcar* / *sinc function* / *impulse train*

with period *0.25 Hz* / *0.5 Hz* / *1 Hz* / *2 Hz* / *4 Hz*.

Figure 3 shows the results of frequency analyses using the Matlab function **pwelch**:
pwelch(signal,rectwin(WINLEN),0,NFFT,Fs,'twosided').

Note that all analyses used a rectangular window with no overlap.

(e) Which plot(s) in Figure 3 show(s) a frequency analysis of $y(t)$?

A / B / C / D / E **circle all that apply**

Justification:

(f) Which plot(s) in Figure 3 show(s) a frequency analysis of $z(t)$?

A / B / C / D / E **circle all that apply**

Justification:

(g) Which plot in Figure 3 was generated using used the shortest window length?

A / B / C / D / E **circle one**

Justification:

(h) All of the analyses in Figure 3 used a rectangular window. How will the plots change if a Hamming window of the same length is used?

spikes will be *wider / narrower / unchanged* **circle one**

average magnitude of regions between spikes will *increase / decrease / stay same* **circle one**

spikes will be *closer together / farther apart / spacing unchanged* **circle one**

Question 2 (15%)

Consider the signal

$$x(t) = P + Q \sin(2\pi Ft) + Rn(t)$$

where

P , Q , and R are constants,

$F = F_1$ for $t < t_1$ sec,

$F = F_2$ for $t > t_1$ sec, and

$n(t)$ is highpass noise above F_H .

$x(t)$ is lowpass filtered with an analog filter with cutoff frequency F_c and sampled at $F_s = 10$ kHz to produce $x[n]$.

(a) Figure 4 shows two spectrograms of $x[n]$. Estimate the values of the following parameters.

$$F_c \approx$$

$$F_1 \approx$$

$$F_2 \approx$$

$$F_H \approx$$

(b) What spectrogram analysis parameter was changed to create the two different plots in Figure 4?

In Questions 3 and 4, lower case letters refer to the sample space description while upper case letters refer to the frequency space description, for example $X[k_1, k_2]$ is the DFT of $x[n_1, n_2]$:

$$\begin{aligned} x[n_1, n_2] &\leftrightarrow X[k_1, k_2] \\ x[n] &\leftrightarrow X[k] \end{aligned}$$

It is **not** necessary to compute the DFT of **any** function in order to answer the questions below.

Question 3 (20%)

Given the following 1-D filters

$$h_1[n] \quad h_2[n] \quad g_1[n] \quad g_2[n]$$

and **separable** 2-D filters defined as follows

$$\begin{aligned} h[n_1, n_2] &= h_1[n_1]h_2[n_2] \\ g[n_1, n_2] &= g_1[n_1]g_2[n_2] \end{aligned}$$

answer parts (a) and (b).

(a) Which of the system block diagrams shown in Figure 5 have a frequency response equivalent to:

$$H[k_1, k_2]G[k_1, k_2]$$

(CIRCLE ALL THAT APPLY)

A / B / C / D / E / NONE

(b)

The system block diagram shown in Figure 6 has a frequency response of:

$$H[k_1, k_2] - G[k_1, k_2]$$

Find expressions for separable filters $a[n_1, n_2]$ and $b[n_1, n_2]$ in terms of $h_1[n_1]$, $g_1[n_1]$, $g_2[n_1]$, $h_2[n_2]$,

$$a[n_1, n_2] = \underline{\hspace{10em}}$$

$$b[n_1, n_2] = \underline{\hspace{10em}}$$

Question 4 (15%)

Consider the 48×48 input image in Figure 7, with values coded from -1 for black to $+1$ for white, as shown. This image contains three values: -1 in the darkest regions, 0 around the border, and 1 in the three light squares. Identify which of the output images in Figure 8 result from applying $h_1[n_1, n_2]$ and $h_2[n_1, n_2]$ to the original input image, where $h_1[n_1, n_2]$ and $h_2[n_1, n_2]$ are defined as:

$$h_1[n_1, n_2] = \begin{matrix} \begin{bmatrix} 0 & -\frac{1}{3} & 0 \\ 0 & -\frac{1}{6} & 0 \\ 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 \\ 0 & \frac{1}{3} & 0 \end{bmatrix} & \begin{matrix} \uparrow \\ n_2 \end{matrix} \\ n_1 \rightarrow \end{matrix} \quad h_2[n_1, n_2] = \begin{matrix} \begin{bmatrix} \frac{1}{25} & \frac{1}{25} & \frac{1}{25} & \frac{1}{25} & \frac{1}{25} \\ \frac{1}{25} & \frac{1}{25} & \frac{1}{25} & \frac{1}{25} & \frac{1}{25} \\ \frac{1}{25} & \frac{1}{25} & \frac{1}{25} & \frac{1}{25} & \frac{1}{25} \\ \frac{1}{25} & \frac{1}{25} & \frac{1}{25} & \frac{1}{25} & \frac{1}{25} \\ \frac{1}{25} & \frac{1}{25} & \frac{1}{25} & \frac{1}{25} & \frac{1}{25} \end{bmatrix} & \begin{matrix} \uparrow \\ n_2 \end{matrix} \\ n_1 \rightarrow \end{matrix}$$

$h_1[n_1, n_2]$: A / B / C / D (circle one)

Justification:

$h_2[n_1, n_2]$: A / B / C / D (circle one)

Justification:

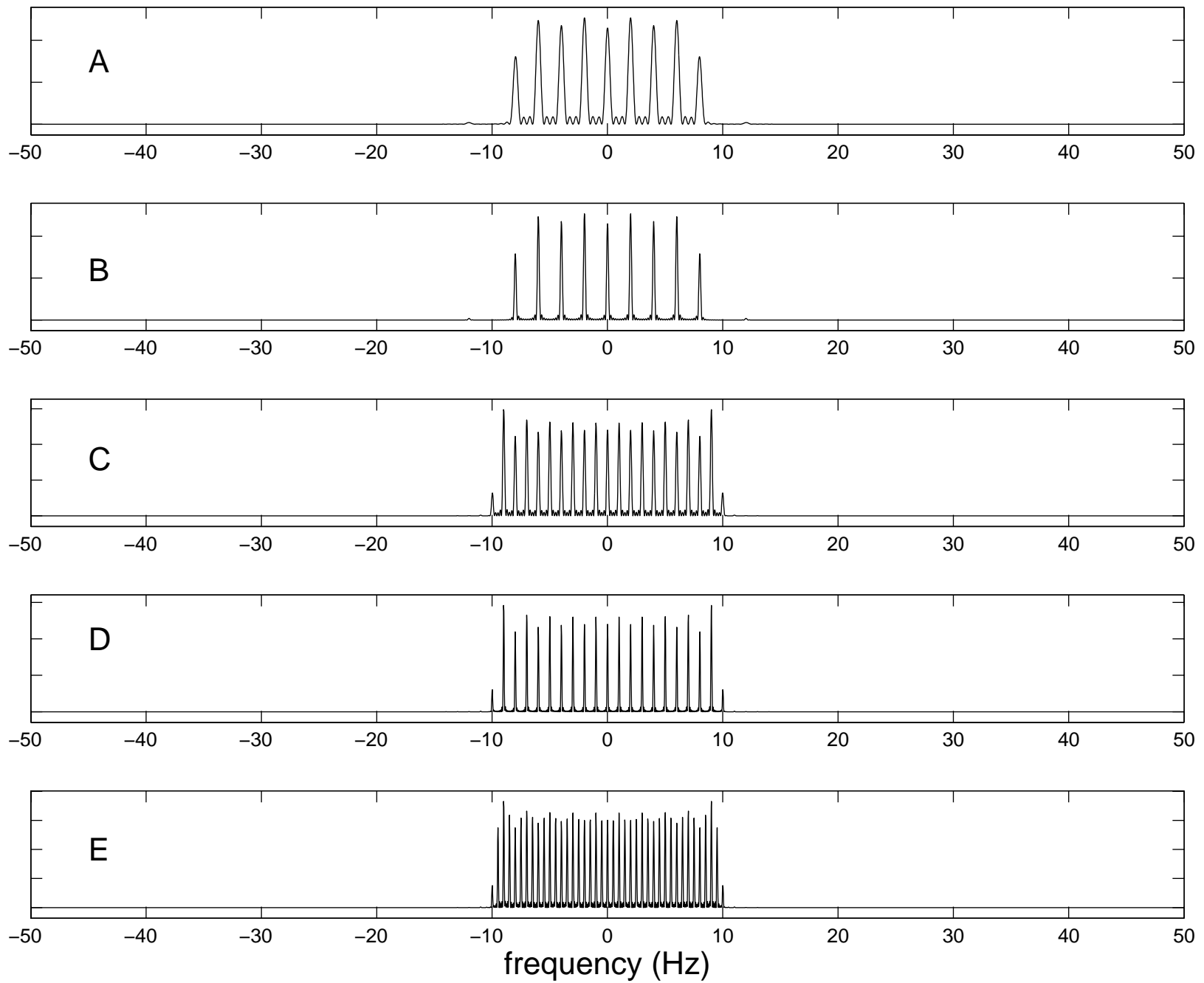


Figure 3:

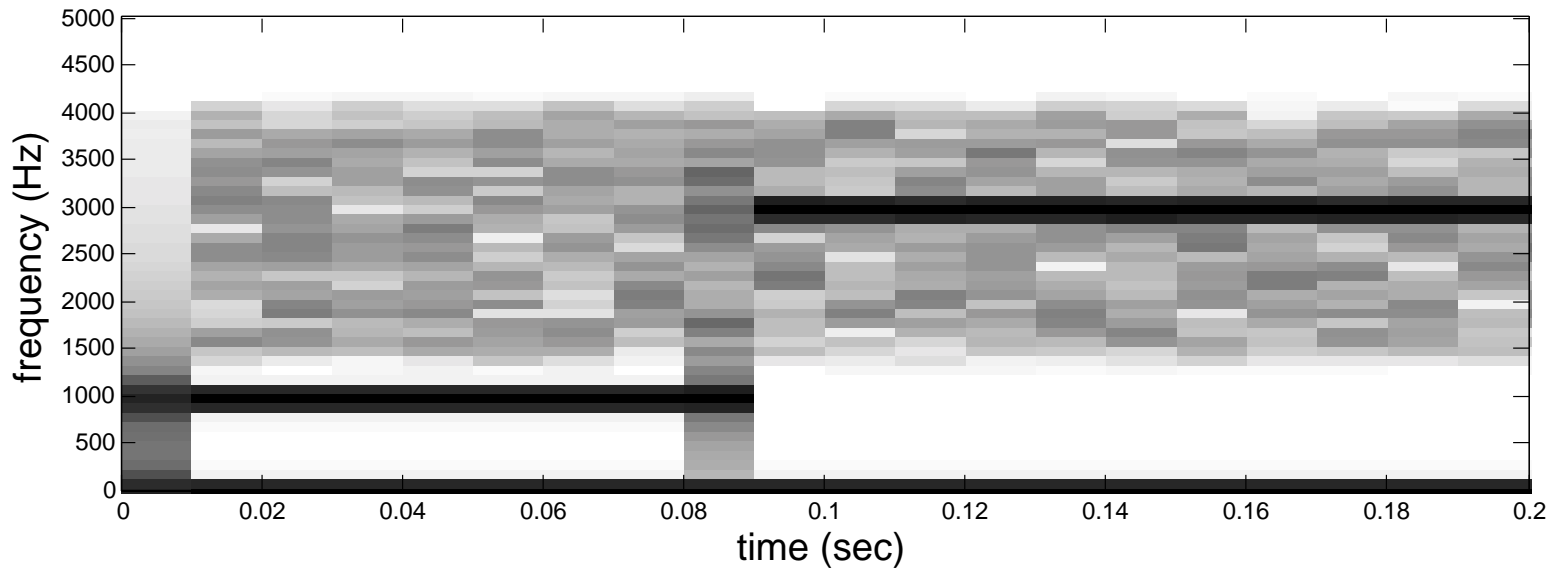
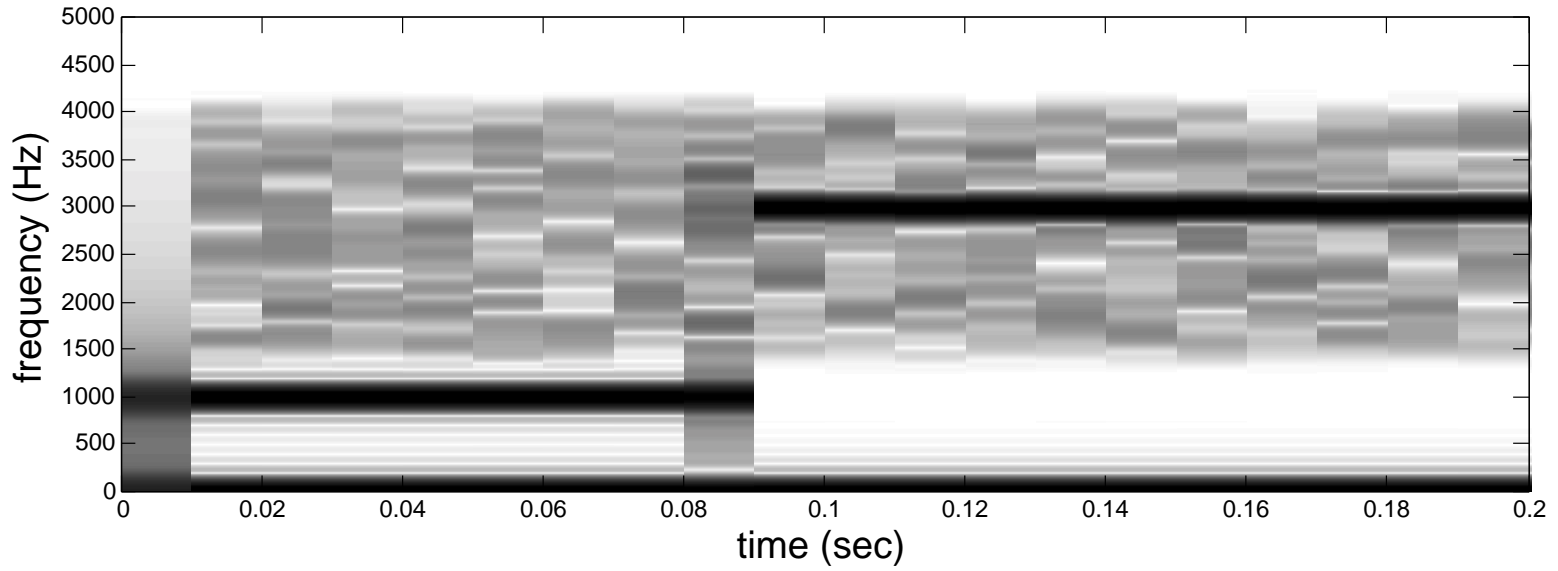


Figure 4:

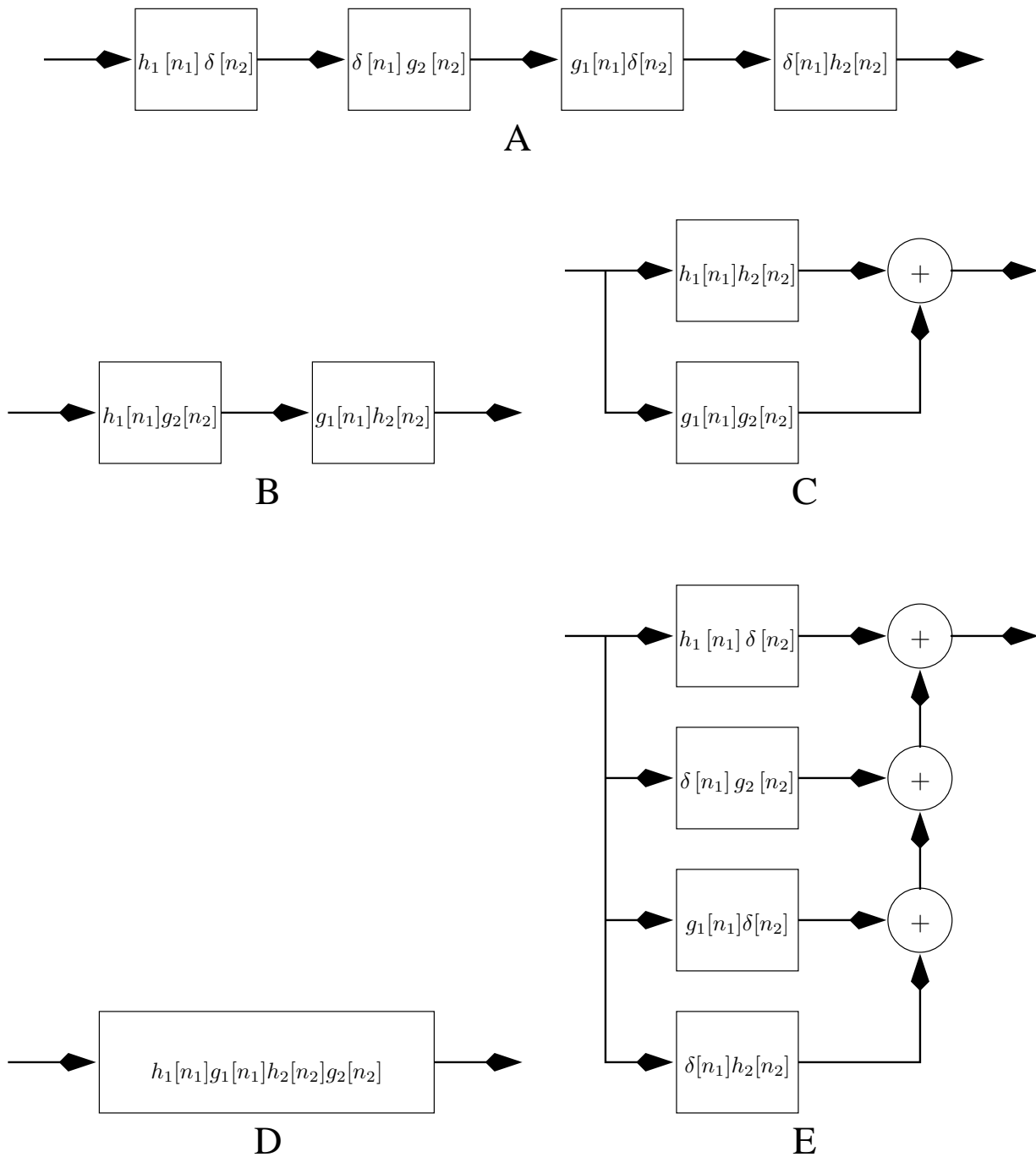


Figure 5: System Block Diagram for Question 3(a).

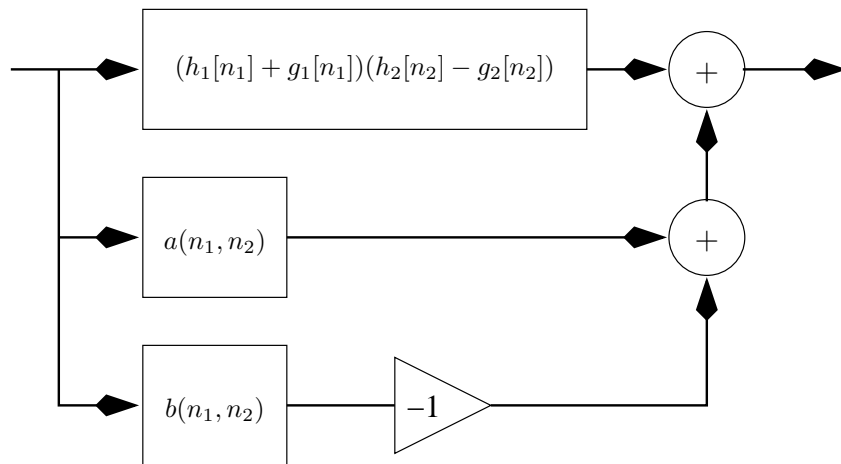


Figure 6: System Block Diagram for Question 3**(b)**.

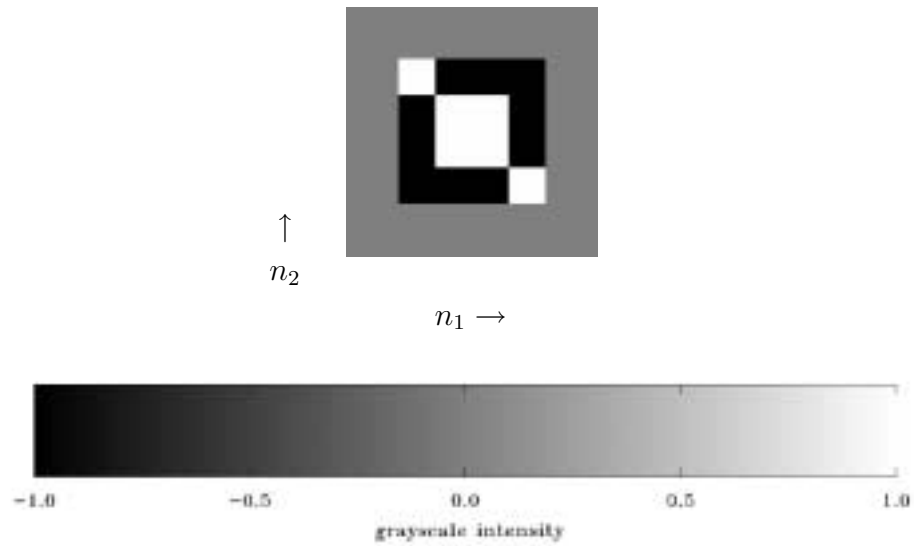


Figure 7: Original image

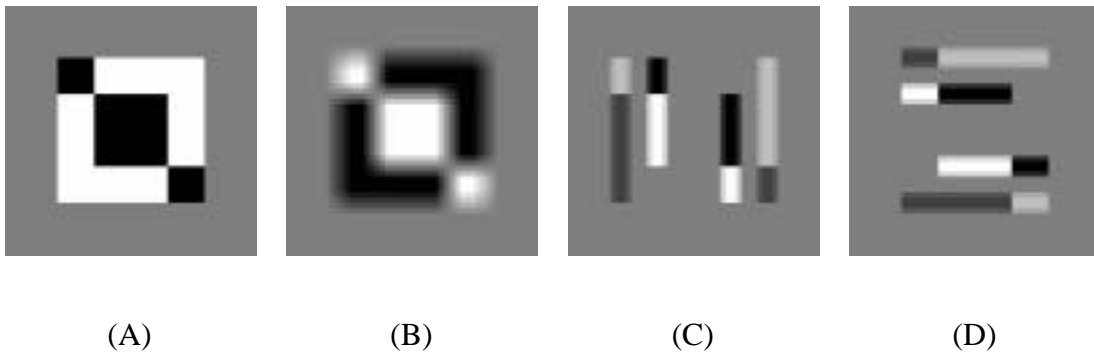


Figure 8: Processed images