

[SQUEAKING]

[RUSTLING]

[CLICKING]

**RICHARD DE NEUFVILLE:** And so what are the conditions for a value function? That is, that the construct of a value function, one that is legitimate in size, meaningful and unambiguous, depends upon three axioms, three assumptions, if you wish, that you might have.

And I'm going to go through them, but by going through them, we also recognize when it is not possible to have a value function. And I want to emphasize that, and this addresses a very important aspect in terms of optimization in particular, but in terms of valuation. There's a large branch of literature which simply does not recognize this reality, and I want to draw your attention to it.

So there are three axioms. The first one is known as Completeness or Complete Preorder, and it basically says that people, in fact, have preferences over all choices. That in no way means that you've thought about all the possible choices, but that you are capable of making a preference. There isn't some thing which you could not decide about.

And deciding about it permits to say that I'm equally attracted to having chocolate or strawberry ice cream, that you don't have to necessarily prefer one to the other, but you can have a valuation. And it's OK to say a dish of strawberry or chocolate ice cream has the same value for me.

It doesn't mean you have to prefer one to the other, but you can make choices. That is for a logical one, that's a necessary assumption. Otherwise, it doesn't mean-- if you can't make preferences, you can't have a function over what happens.

The second one is transitivity. That is if thing  $x_1$  is preferred to thing  $x_2$ , and thing  $x_2$  is preferred to thing  $x_3$ , then the preference is transitive, that the  $x_1$  is better than  $x_2$ , which is better than  $x_3$ . Therefore,  $x_1$  is preferred to  $x_3$ .

Now, it's assumed true for individuals, and it is true that depending on how you phrase it, there are various ways you can trick people into thinking differently about it, depending on what else-- the context you put in, their known examples of it. But this is a fairly standard one.

If you're a strawberry ice cream fan, and you prefer it to chocolate, and you really hate-- I don't know-- marshmallow ice cream, then it's presumed that you are preferring strawberry to marshmallow also. So this is a normal thing.

The trick here that I want to bring to your mind, and we'll explore it in a moment, is that this fairly obvious choice of A being preferred to B being preferred to C, therefore A is preferred to C, that is not necessarily true for groups.

So whereas we can have a meaningful value function for individuals, having it for groups is not very plausible. That is, and when I talk about groups, I mean, we're talking about a board of directors. We're talking about a design team. We're talking about a city council. We're talking about almost anything that you can imagine where you have individuals or factions or business groups talking about things.

Right now, I'm dealing with a company in Morocco, and as with others, getting decisions made is a tug of war between the finance people, the marketing people, the industrial people each have a different set of values as what makes sense from their point of view. And they do not have a consistent value function for decisions about what to spend money on or how to run the company. But that's the transitivity one. Yes?

**AUDIENCE:** Yeah, so I just had a comment on transitivity. So these are some of Savage's axioms, I'm assuming, for subjective expected utility. But the transitivity one, there was this experiment that was done, and it's called the Ellsberg paradox, that shows that under ambiguity, transitivity doesn't always hold because people will want to choose the non-ambiguous option usually.

**RICHARD DE NEUFVILLE:** Yes. Yes, point well taken. And there are various ways that, as you say, under various ambiguous situations, and there's also the Allais Paradox, and there's a cluster of psychological experiments where by asking a question that context is something else, you can get people to focus on different aspects. So you can-- there's a range of exceptions.

But in general, this is an acceptable one. And my point here is not so much to say that it is-- there aren't exceptions. It is that in this case, it's relatively OK, if you had to think about your own decisions. But when we get to groups, it is definitely not OK. That's the point I want to emphasize in this context.

So well done from an advanced doctoral student who studied this, and you're absolutely correct. But for the-- let's-- yeah, let me just leave it at that. Well said. I appreciate your comment. It's excellent. Good going.

So the third one is sometimes called the Archimedean Principle or the Monotonicity, but it basically says that if we are looking between a range, which is the thing up top between the best of something or the most of something and the worst or something, for anything in between, that the value of the thing in between is a weighting function-- that is, a sum proportion such that the value of the thing in between is a certain proportion of the best and a complimentary proportion of the worst.

It basically says that the valuation may be going up all the time, or it may be going down all the time, but it's not going up and then down or up and down and up. That is, it is basically monotonic so that you can think about something being better than something and some else thing being better. If you don't have that, you can't be transitive.

So the question is then, well, OK, is this a reasonable thing? Because if you want a consistent way to value something-- so if you're saying I'm saving lives or making money, that more and more is always better or less, less, less is always better, that there isn't a point where you say, well, it's good to have so much. But then if I have more money, that's really bad. Or if I've saved a lot of lives, but if I save more, it's really bad or worse.

So is it a reasonable assumption? I mean, it's a necessary assumption for a consistent value function, but it is reasonable. So no, it's not always true, is the reason I bring it up. And let's continue on here. Let's look at some example cases. If I have sound from a speaker, I want it loud enough so I can hear it. But at some point, if the sound got too loud, maybe it's just because I'm older than the youngest people might like it really booming, but at some point, it's just too loud for me. I can't do it.

Do I like salt in my food or sugar in my coffee or whatever? Well, more is better up to a point, until more is worse. So the point being this valuation can be very complicated. Now, we get around this in many cases by saying, say to the salt on the food, we can say, all right, I don't mind having more salt.

It's always better to have more salt on the table, I suppose. But I don't have to use it. So having it available is not the same thing as having to use it. So in that case, you can assume-- you can think through the problem as saying, well, at a certain point, I don't benefit anymore from it, so it levels off, but it's not worse to have it available.

So there are ways around this particular thing. But what I want to emphasize here, that there is this Monotonicity assumption, and it doesn't always apply unless you work your way around it in some way.