Homework 3: Decision Analysis and EVPI
IDS.332 and IDS.333

Concept of this assignment
The purpose of the two problems in this assignment is to
- validate your understanding of how to set up a decision tree,
- determine the best choice, and
- calculate the expected value of perfect information (EVPI) as upper limit on value of new information.

This assignment prepares you for the end of class quiz, which will include similar questions.

Problem 1

Traffic Department

As head of the Traffic Department you are planning a new system of traffic lights. Your experts are divided into two groups. One believes in theory A and one in theory B (A or B is true, not both). At present you consider the two theories equally plausible. You have two options for your light system—systems X and Y.

　　　If theory A is true and you adopt system X, then your payoff will be 1 with probability 0.8, and 0 with probability 0.2. If theory B is true and you adopt system X, then your payoff will be 1 with probability 0.1, and 0 with probability 0.9. Your payoff to system Y is 0.5.

　　　You have a two-period time horizon. You can choose system Y now for the two periods or you can experiment for the first period (i.e., use system X) and then choose a system for the second period depending on your payoff in period 1.

(*a*) Draw the decision tree. Label carefully.
(*b*) Put the payoffs at the ends of the branches.
(*c*) Put the probabilities you can write down immediately on the appropriate branches.
(*d*) Explain how to compute the other probabilities.
(*e*) Assuming that you wish to maximize the expected value of the payoff, what strategy should you follow?

The reasoning to follow is:
1) the observable outcome is whether 1 or 0 occurs after X – not if A or B is true.
2) these outcomes can come from either A or B. We can calculate probability of observing 1 as ½ (0.8) + ½ (0.1) = 0.45. P(zero) = 0.55, the complement.
3) notice that for a single period that EV(X) =0.45 < EV(Y) = 0.5. – BUT running X provides information; it changes our perception of likelihood of A or B. Specifically, if we observe 1, it's much more likely that A is true, and that choosing X again has an EV much higher than 0.45.
4) The preferred strategy is thus likely to be: Choose X for first period, then if you get 1 choose X again (EV ~0.8, say – could use Bayes' Theorem); if you get 0, go with Y to get ½. The next result is likely to outperform the sure thing.
The overall takeaway: Information has value and Experimentation can be best

Problem 2.

Marian Haste is a painter under contract to paint the exterior of a house for $50,000. Unfortunately for her, the outside temperature may drop below freezing. If it does, the paint will not stick well and may peel off. The paint may also peel off even if it doesn't freeze. Either way the peeling happens, Marian would have to repaint the house, at a cost to her of $30,000.

The weather forecast indicates there is a 60% chance of rain. Marian also believes there is a 1/3 chance the paint may peel if the rain freezes, but only a 10% chance if it does not. Her choices now are either to go ahead and paint, or to defer the until it's warm enough so it won't possibly freeze. Deferring the job would require her to pay $6,000 in overtime and penalties.

- What would you advise Marian to do? (Should Marian Haste go ahead, and repaint at leisure?)
- What is the most Marian should pay for better information about the weather?

The issue here concerns "peeling" and its probability
This can occur either in bad weather 1/3 (60%) = 0.20
Or bad application, 0.1 (40%) = 0.04
Total = 0.24
EV (paint now) = 0.24 (50-30) + 0.76(50) = 4.8 + 38 = 42.8
EV (delay) = 50 – 8 = 42
   a) Without weather information, EV (paint now) is greater than EV (delay). Hence, Marian should paint now.

For EVPI
If say Rain (p= 0.6) you defer and get 42
If say "no Rain (p=0.4) you paint now and get 47
Expected value after Perfect Info = 0.6(42) + 0.4 (47) ~ 25.2 + 18.8 = 44
EVPI = 44 – 42.8 = 1.2
Maybe Marian would be justified in paying somewhat over $500, perhaps $600…

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