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Multidisciplinary System Design Optimization (MSDO)

Decomposition and Coupling Lecture 4

Anas Alfaris



- Information Flow and Coupling
- MDO frameworks
 - Single-Level (Distributed analysis)
 - Multi-Level (Distributed design)
 - Collaborative Optimization
 - Analytical Target Cascading
 - (Hierarchical Decomposition & Multi-Domain Formulation)

Mesd Standard Optimization Problem

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That is, find x^* s.t. $J(x^*) \le f(x)$, $\forall x \in \text{dom}(J) \cap \text{dom}(g)$ © Massachusetts Institute of Technology - Prof. de Weck and Prof. Willcox





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Information Flow







Information Flow





Advantages of Decoupling



Computation of g(x) can be very time consuming, want to divide the work and compute in parallel. For example, if $x = (x_1, x_2)$, where $x_1 \in \mathbb{R}^{n1}$, $x_2 \in \mathbb{R}^{n2}$ and $g(x) = (g_1(x_1), g_2(x_2))$

Then g_1 and g_2 can be computed in parallel. Graphically,











The decoupled constraints assumption is not general. Subsystems can be coupled and loops can arise. For example,





x: decision variables
w: SS outputs (constraint, cost)
u: SS input (dependent)

vline: SS input hline: SS output

Computation of w_1 and w_2 requires an iterative method. © Massachusetts Institute of Technology - Prof. de Weck and Prof. Willcox



Coupling



• An example where such a loop happens is as follows: $\min J(x_1, x_2)$ $w_1 = g_1(x_1, g_2(x_2, w_1)) \ge 0$

s.t. $w_1 = g_1(x_1, g_2(x_2, w_1)) \ge 0$ $w_2 = g_2(x_2, g_1(x_1, w_2)) \ge 0$

where $x_1 \in \mathbb{R}^{n_1}, x_2 \in \mathbb{R}^{n_2}, g_i : x_i \times u_i \mapsto w_i, i = 1, 2$

*w*₁ and *w*₂ satisfy coupled relations at each optimization iteration. At each constraint evaluation, nonlinear equations must be solved (e.g. by Newton's method) in order to obtain *w*₁ and *w*₂, which can be time consuming.

Want a way to return to the situation of decoupled constraints.

Mesd Surrogate Variables ("Tearing")

Information loop can be broken by introducing surrogate variables.

min
$$J(x_1, x_2)$$

s.t. $w_1 = g_1(x_1, g_2(x_2, w_1)) \ge 0$
 $w_2 = g_2(x_2, g_1(x_1, w_2)) \ge 0$

min $J(x_1, x_2)$ s.t. $g_1(x_1, u_1) \ge 0$ $g_2(x_2, u_2) \ge 0$ $u_2 - g_1(x_1, u_1) = 0$ $u_1 - g_2(x_2, u_2) = 0$ 16 **8**88

• u_1 and u_2 are decision variables acting as the inputs to g1(SS1) and g2 (SS2). Introducing surrogate variables breaks information loop but increases the number of decision variables.



Numerical Example

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 $\min J_1 + J_2$ $\min x_1^2 + x_2^2 + (x_3 - 3)^2 + (x_4 - 4)^2$ decoupled s.t. $w_1 \ge 0$ s.t. $w_1 = x_1^3 - x_2^3 + 2x_5 \ge 0$ $w_2 \ge 0$ $w_2 = x_3^3 - x_4^3 + 2x_6 \ge 0$ where $J_1 = x_1^2 + x_2^2$ $x_1^3 - x_2^3 + 2x_5 - x_6 = 0$ $J_2 = (x_3 - 3)^2 + (x_4 - 4)^2$ $x_3^3 - x_4^3 + 2x_6 - x_5 = 0$ $w_1 = x_1^3 - x_2^3 + 2w_2$ $w_2 = x_3^3 - x_4^3 + 2w_1$ Solution: coupled $x = (0, 0, 4, 3, 12\frac{1}{3}, 24\frac{2}{3})$ MATLAB® 5.3 $\min x_1^2 + x_2^2 + (x_3 - 3)^2 + (x_4 - 4)^2$ s.t. $w_1 = g_1(x_1, x_2, x_3, x_4) \ge 0$ coupled: 356,423 FLOPS 4.844s uncoupled: 281,379 FLOPS 0.453s $w_2 = g_2(x_1, x_2, x_3, x_4) \ge 0$

esd Single-level and Multi-Level Frameworks

Single-level (Distributed Analysis)

-disciplinary models provide analysis -all optimization done at system level

Multi-level (Distributed Design)

-provide disciplinary models with design tasks -optimization at subsystem and system levels non-hierarchical decomposition

hierarchical decomposition





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- Disciplinary models provide analysis
- Optimization is controlled by some overseeing code or database
 - *e.g.* ISight (Optimizer)



Single-level (Distributed Analysis)

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- During the optimization, the overseeing code keeps track of the values of the design variables and objective
- The values of the design variables are changed according to the optimization algorithm
- Disciplinary models are asked to evaluate constraints/objective



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- Multi-level Optimization methods distribute decision making throughout the system
- Subsystem level models are provided with design tasks
- Optimization is performed at a subsystem level in addition to the system level
- Provide some autonomy to design groups and reduces communication requirements.









Collaborative Optimization (CO)

- disciplinary teams satisfy local constraints while trying to match target values specified by a system coordinator
- preserves disciplinary-level design freedom.
- CO is used typically to solve discipline-based decomposed system optimization problems.





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Mesd Collaborative Optimization

Two levels of optimization:

- A system-level optimizer provides a set of targets.
 - These targets are chosen to optimize the system-level objective function

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• A subsystem optimizer finds a design that minimizes the difference between current states and the targets.

- Subject to local constraints

Collaborative Optimization

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CO – Subsystem Level

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- The subsystem optimizer modifies local variables to achieve the best design for which the set of local variables and computed results most nearly matches the system targets
- The local constraints must also be satisfied



CO – System Level



min J_{sys} wrt: $\mathbf{x}_0 =$ target variables s.t. $J_k = 0 \quad \forall$ subproblems_k

- System-level optimizer changes target variables to improve objective and reduce differences J_k
 - $-J_k=0$ are called compatibility constraints
 - compatibility constraints are driven to zero, but may be violated during the optimization

Mesd CO Example: Aircraft Design

Consider a simple aircraft design problem: maximize range for a given take-off weight by choosing wing area, aspect ratio, twist angle, L/D, and wing weight.

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Mesd CO Example: Aircraft Design



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- \mathbf{x}_0 = system-level target variable values
- **x** = subsystem local variables
- \mathbf{y}_{ij} = coupling functions

- y_{ij} =outputs of subsystem *j* which are needed as inputs to subsystem *i*.
- Coupling equations must also be satisfied, so coupling variables are included in subsystem objective.
- Used to reduce the number of system-level parameters.

Mesd Analytical Target Cascading



• ATC was initially developed as a product development tool to cascade system-level product targets through a hierarchy of design groups

• ATC is typically used to solve object-based decomposed system optimization problems

• The ATC paradigm is based on hierarchical organizational and analysis structures

• ATC approach is to take a high-level system analysis and use more detailed subsystem analyses at the lower levels.

esd Analytical Target Cascading



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Mesd Analytical Target Cascading



• ATC's mathematical formulation is similar to CO although they were developed with different motivations.

• Bottom level problems have the most design freedom. Many possible solutions can exist that both match targets while satisfying local design constraints.

• At higher levels design freedom is progressively reduced, until it is a minimum at the top level.

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Analytical Target Cascading

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Top Level Problem

 \mathbf{P}_{sup}

 $\min_{\mathbf{x}^{\dagger}_{sup} = \{\mathbf{x}_{sup}, \mathbf{y}^{\dagger}_{s}, \mathbf{R}_{s}, \varepsilon_{R}, \varepsilon_{y}\}}$

subject to

$$egin{aligned} &||\mathbf{R}_{sup} - \mathbf{T}_{sup}|| + arepsilon_R + arepsilon_y \ &\sum_{k \in C_{sup}} ||\mathbf{R}_{s,k} - \mathbf{R}_{s,k}^L|| \leq arepsilon_R \ &\sum_{k \in C_{sup}} ||\mathbf{y}^\dagger_{s,k} - \mathbf{y}^\dagger_{s,k}^L|| \leq arepsilon_y \ &g_{sup}(\mathbf{x}_{sup}, \ \mathbf{R}_s) \leq 0 \end{aligned}$$

 $h_{sup}(\mathbf{x}_{sup}, \mathbf{R}_s) \leq 0$

Intermediate Level Problem $\mathbf{P}_{s,j}$

 $\min_{\mathbf{x}^{\dagger}_{s,j} = \{\mathbf{x}_{s,j}, \mathbf{y}^{\dagger}_{s,j}, \mathbf{y}^{\dagger}_{ss}, \mathbf{R}_{ss}, \varepsilon_R, \varepsilon_y\} }$ subject to

$$egin{aligned} &||\mathbf{R}_{s,j}-\mathbf{R}_{s,j}^U||+||\mathbf{y}_{s,j}-\mathbf{y}_{s,j}^U||+arepsilon_R+arepsilon_y\ &\sum_{k\in C_{s,j}}||\mathbf{R}_{ss,k}-\mathbf{R}_{ss,k}^L||\leqarepsilon_R\ &\sum_{k\in C_{s,j}}||\mathbf{y}^\dagger_{ss,k}-\mathbf{y}^\dagger_{ss,k}||\leqarepsilon_y\ &g_{s,j}(\mathbf{x}_{s,j},\ \mathbf{y}^\dagger_{s,j},\ \mathbf{R}_{s,j})\leq 0\ &h_{s,j}(\mathbf{x}_{s,j},\ \mathbf{y}^\dagger_{s,j},\ \mathbf{R}_{s,j})\leq 0 \end{aligned}$$

Bottom Level Problem $\mathbf{P}_{ss,j}$

$$\begin{split} \min_{\mathbf{x}^{\dagger}_{ss,j} = \{\mathbf{x}_{ss,j}, \mathbf{y}^{\dagger}_{ss,j}\}} & \qquad ||\mathbf{R}_{ss,j} - \mathbf{R}_{ss,j}^{U}|| + ||\mathbf{y}_{ss,j} - \mathbf{y}_{ss,j}^{U}|| \\ & \qquad \text{subject to} & \qquad g_{s,j}(\mathbf{x}_{s,j}, \ \mathbf{y}^{\dagger}_{s,j}, \ \mathbf{R}_{s,j}) \leq 0 \\ & \qquad h_{s,j}(\mathbf{x}_{s,j}, \ \mathbf{y}^{\dagger}_{s,j}, \ \mathbf{R}_{s,j}) \leq 0 \end{split}$$

Mesd Analytical Target Cascading



- Linking variables *y*: Quantities that are input to more than one subspace. These could be either shared variables or coupling variables.
- Local decision variables *x*: Variables that a particular subspace determines the value of.
- Responses *R*: Values generated by subspaces required as inputs to respective parent subspaces.
- Targets *T*: Values set by parent subspaces to be matched by the corresponding quantities from child subspaces.
- ER and Ey: allowable compatibility tolerance.



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Decomposition



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Decomposition

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Decomposition Building System



	System	Туре	# of FPs	# of BPs
	Building	Mode	11	14
e	Building	Network	12	8
le l	Wator	Mode	19	10
Fe	vvater	Network	17	13
	From	Mode	21	10
0	Energy	Network	14	13
	Transportation	Mode	18	16
	transportation	Network	21	11

	20	
Object	Aggregation	
Process(ing)	Exhibition	Effect
	Generalization	

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Formulation



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Formulation



	0	0	0	0	0		0	0	0	0	0		0	0	0	0	0
	1	0	0	0	0		0	0	0	0	0		0	0	0	0	0
A =	1	0	0	0	0	$A^2 =$	0	0	0	0	0	$A^3 =$	0	0	0	0	0
	0	0	1	0	0		1	0	0	0	0		0	0	0	0	0
	0	1	0	0	0		1	0	0	0	0		0	0	0	0	0

Where:

 π_j is an indicator of the fraction of total elements to which element *j* provides input,

 δ_i is the fraction of total elements on which element *i* depends, and \mathbf{a}_{ij} is an element of a matrix that can be the DSM, a power of the DSM, or the V matrix.





Formulation



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Formulation

Parameters Set for Cycle 1

BS - 1 Cell Type Assignment
BS-2 Height
BS - 3 Occupancy Level
BS - 4 Specifications
BS - 6 Land Use/System Building Generation
RS . 7 Water Use/System Building Generation
PC 9 Energy Hea/Sustam Publican Constration
BC 0 Transportation Line Custom Pullding Concention
DS - 9 Transportation UserSystem building Generation
BS - 11 Architectural Dependency
BS - 12 Zone Type Assignement
Tr - 1 Speed (Mode & Network)
Tr - 2 Specifications (Mode)
Tr - 3 Avg System Life (Mode)
Tr - 4 Avg Travel Distance (Mode & Network)
Tr - 5 # of travellers/trip (capacity) Mode
Tr - 6 # of cars / Frequency
Tr - 7 Reliability of System (Mode & Network)
Tr - 8 Cost/Trip or Distance (Mode)
Tr - 9 Emissions per unit (Mode)
Tr - 14 Avg System Life (Network)
Tr - 15 Guidance (Network)
Tr. 16 Specifications (Network)
III - To Specifications (Network)
Tr-17 Level
Tr - 17 Level Tr - 18 Station Locations
Tr - 17 Evel Tr - 18 Station Locations Tr - 19 Stations Size
Tr - 17 Level Tr - 18 Station Locations Tr - 19 Stations Size Tr - 10 Land UselSystem (Mode)
Tr - 17 Level Tr - 18 Station Locations Tr - 19 Stations Size Tr - 10 Land Use/System (Mode) BS - 22 Total Area Generated/Type
Tr - 17 Level Tr - 18 Station Locations Tr - 19 Stations Size Tr - 10 Land Use/System (Mode) BS - 22 Total Area Generated/Type BS - 23 # of Floors Generated
Tr - 17 Level Tr - 18 Station Locations Tr - 19 Stations Size Tr - 10 Land Use/System (Mode) BS - 22 Total Area Generated/Type BS - 23 # of Floors Generated SS - 23 # of Floors Generated SS - 26 Land Cons./Demand Building Generation
Tr-17 Level Tr-18 Station Locations Tr-19 Stations Size Tr-10 Land Use/System (Mode) BS-22 Total Area Generated/Type BS-23 ≢ of Filors Generated BS-26 Land Cons./Demand Building Generation BS-17 Denstry at locations
Tr-17 Level Tr-18 Station Locations Tr-19 Stations Size Tr-10 Land Use/System (Mode) BS-22 Total Area Generated/Type BS-23 # of Floors Generated BS-26 Land Cons./Demand Building Generation BS-17 Density at locations BS-18 Total Occupants / Bidg Type
Tr - 17 Evel Tr - 18 Station Locations Tr - 19 Stations Size Tr - 10 Land Use/System (Mode) BS - 22 Total Area Generated/Type BS - 23 # of Floors Generated BS - 26 Land Cons./Demand Building Generation BS - 17 Density at locations BS - 17 Total Occupants / Bidg Type BS - 27 Water Cons./Demand Building Generation
Tr - 12 Level Tr - 13 Station Locations Tr - 19 Stations Size Tr - 10 Land Use/System (Mode) BS - 22 Total Area Generated/Type BS - 23 # of Floors Generated BS - 26 Land Cons./Demand Building Generation BS - 17 Density at locations BS - 18 Total Occupants / Bidg Type BS - 27 Water Cons./Demand Building Generation BS - 28 Leneroy Cons./Demand Building Generation BS - 28 Leneroy Cons./Demand Building Generation BS - 28 Leneroy Cons./Demand Building Generation
Tr - 17 Level Tr - 18 Station Locations Tr - 19 Stations Size Tr - 10 Land Use/System (Mode) BS - 22 Total Area Generated/Type BS - 23 # of Floors Generated SS - 26 Land Cons./Demand Building Generation BS - 17 Density at locatons BS - 18 Total Occupants / Bidg Type BS - 27 Water Cons./Demand Building Generation BS - 28 Energy Cons./Demand Building Generation BS - 28 Energy Cons./Demand Building Generation BS - 29 Trans. Cons./Demand Building Generation
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Tr-17. Level Tr-18. Station Locations Tr-19. Station Size Tr-10. Land Use/System (Mode) BS-22. Total Area Generated/Type BS-23. ≢ of Filors Generated BS-26. Land Cons./Demand Building Generation BS-17. Densty at locations BS-18. Total Occupants / Bidg Type BS-27. Water Cons./Demand Building Generation BS-28. Energy Cons./Demand Building Generation BS-29. Trans. Cons./Demand Building Generation BS-29. Trans. Cons./Demand Building Generation BS-29. Trans. Cons./Demand Building Generation BS-29. Trans. Cons./Demand Building Generation BS-34. Cluster Size BS-35. Logical Attraction
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Tr - 17 Level Tr - 18 Station Locations Tr - 19 Stations Size Tr - 10 Land Use/System (Mode) BS - 22 Total Area Generated/Type BS - 23 # of Floors Generated BS - 26 Land Cons./Demand Building Generation BS - 17 Density at locations BS - 18 Total Occupants / Bidg Type BS - 27 Water Cons./Demand Building Generation BS - 28 Trans. Cons./Demand Building Generation BS - 29 Trans. Cons./Demand Building Generation BS - 34 Cluster Size BS - 35 Logical Attraction BS - 36 Architectural Attraction BS - 36 Architectural Attraction BS - 36 Architectural Attraction BS - 36 Logical Repuision
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Tr - 17 Level Tr - 18 Station Locations Tr - 19 Stations Size Tr - 10 Land Use/System (Mode) BS - 22 Total Area Generated/Type BS - 23 # of Floors Generated BS - 26 Land Cons./Demand Building Generation BS - 17 Density at locatons BS - 18 Total Occupants / Bidg Type BS - 27 Water Cons./Demand Building Generation BS - 28 Energy Cons./Demand Building Generation BS - 29 Trans. Cons./Demand Building Generation BS - 32 Lugical Attraction BS - 36 Architectural Attraction BS - 38 Achitectural Repulsion BS - 39 Architectural Repulsion Tr - 27 Number of Stations
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Tr - 17. Level Tr - 18. Station Locations Tr - 19. Stations Size Tr - 10. Land Use/System (Mode) S 22. Total Area Generated/Type BS - 22. Total Area Generated/Type BS - 23. # of Floors Generated BS - 26. Land Cons./Demand Building Generation BS - 17. Density at locations BS - 18. Total Occupants / Bidg Type S 27. Water Cons./Demand Building Generation BS - 28. Land Cons./Demand Building Generation BS - 29. Trans. Cons./Demand Building Generation BS - 29. Trans. Cons./Demand Building Generation BS - 29. Trans. Cons./Demand Building Generation BS - 34. Cluster Size BS - 35. Logical Attraction BS - 36. Architectural Attraction BS - 39. Architectural Repulsion Tr - 27. Number of Stations Tr - 28. Average Travel Time Tr - 32. Avg Waiking Distance



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