24.118 – Paradox and Infinity
Problem Set 5: Infinity

How this problem set will be graded:
Unlike the previous problem set, this time you will be graded both on the
basis of whether your answers are correct and on the basis of whether they are
properly justified. There is no word limit.
As with all problem sets, you may consult published literature and the web.
You must, however, credit all sources. Failure to do so constitutes plagiarism
and can have serious consequences. For advice about when and how to credit
sources see:


Problems:

1. Show that there is a 1-1 correspondence between the whole numbers (\ldots -3, -2, -1, 0,
   1, 2, 3, \ldots) and the prime numbers.

2. Show that there is a 1-1 correspondence between the natural numbers and the set of
   all pairs \((n, m)\) for \(n\) and \(m\) natural numbers.

3. Show that there is a 1-1 correspondence between \([0,1)\) and \([0,\infty)\) (i.e. between the
   set of real numbers greater or equal to zero and smaller than 1, and the set of real
   numbers greater or equal to 0.)

4. Show that there can be no 1-1 correspondence between the natural numbers and the
   set of all functions from the natural numbers to the set \(\{a, b, c, d, e\}\).

5. Consider the following infinite tree:

\[
\begin{array}{c}
0 \\
/ \\
0 1 \\
/ / \\
0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1
(When fully spelled out, the tree contains one row for each natural number. The zero-th row contains one node, the first row contains two nodes, the second row contains four nodes, and, in general, the nth row contains $2^n$ nodes.)

Is there a 1-1 correspondence between the nodes of this tree and the natural numbers? Is there a 1-1 correspondence between the branches\footnote{A branch is an infinite sequence of nodes which starts at the top of the tree and contains a node at every row, with each node connected to its successor by an edge. (Branches can be represented as infinite sequences of zeroes and ones.)} of this tree and the natural numbers? (Don’t forget to justify your answers!)
24.118 Paradox & Infinity
Spring 2013

For information about citing these materials or our Terms of Use, visit: http://ocw.mit.edu/terms.