The story we’ve been telling has it that a name denotes an individual and an open sentence represents a property, and the sentence obtained by substituting a name for free occurrences of a variable in an open sentence is true if and only if the individual designated by the name has the property. That can’t be all there is to the story, as the following example shows: The *Waverley* novels were published anonymously, and their authorship was a matter of lively speculation among the British ruling classes. The following sentence was true:

The king wondered whether Scott was the author of *Waverley*.

According to the account we’ve been developing, the name “Scott” denotes an individual, the open sentence “The king wondered whether x was the author of *Waverley*” designates a property, and the sentence is true because the individual named has the property. Now the individual named by “Scott” is the individual names by “the author of *Waverley*.” Hence, the individual named by “the author of *Waverley*” has the property designated by the open sentence “The king wondered whether x was the author of *Waverley*.” Therefore, according to the story we’ve been developing,

The king wondered whether the author of *Waverley* was the author of *Waverley* should be true. But that’s absurd. The king never had the slightest doubt that the author of *Waverley* was the author of *Waverley*.

Again, “Phosphorus” is an old name for the Morning Star, and “Hesperus” an old name for the Evening Star. One of the earliest astronomical discoveries was that Phosphorus and Hesperus, were, in fact, the same. We have:

That Phosphorus = Hesperus was an important discovery of ancient astronomers.

According to our semantics, this means that the planet designated by “Phosphorus” has the property designated by the open sentence “That x = Hesperus was an important discovery of ancient astronomers.” Now any property had by Phosphorus is also had by Hesperus, so the planet designated by “Hesperus” has the property designated by “That x = Hesperus was an important discovery of ancient astronomers.” Hence:

That Hesperus = Hesperus was an important discovery of ancient astronomers.

But that’s silly. It didn’t take an astronomer to realize that Hesperus = Hesperus.

We are finding exceptions to the rule SI, as it is applied to English. Another example,

Necessarily, 9 is a perfect square.
9 = the number of planets.
Therefore, necessarily, the number of planets is a perfect square.
Again,

Hegel believed that the number of planets was prime.
9 = the number of planets.
Therefore, Hegel believed that 9 was prime.

Frege proposed to distinguish the sense of an expression from its reference. The sense of an expression is what the expression means, whereas its reference is that the expression refers to. You can know what the phrase “the number of planets” means without knowing what number it refers to. For that, you require astronomical knowledge as well as linguistic competence.

The sense of a sentence is a proposition. The reference of a declarative sentence, according to Frege, is its truth value, either true or false.

Most of the time, a phrase refers directly, to whatever it refers to. Sometimes, however, the phrase refers obliquely, and the oblique reference of a phrase is its usual sense. Oblique reference occurs within mental attitude reports, after such phrases as “believes that,” “suspects that,” “denies that,” “knows that,” “desires that,” “learned that,” and “wonders whether.” It also occurs in indirect speech reports (“said that”) and after modal terms like “necessarily,” “possible,” and “probably.”

In direct contexts, the numeral “9” refers to a number, but in oblique contexts it refers to the sense of a numeral. What the numeral “9” refers to in “Hegel believed that the 9 is a prime number” is different from what the phrase “the number of planets” refers to in “Hegel believed that the number of planets is a prime number,” even though what “9” refers to in ordinary contexts is what “the number of planets” refers to in ordinary contexts. Thus the inference

Hegel believed that the number of planets is prime.
9 = the number of planets.
Therefore, Hegel believed that 9 is prime.

commits a fallacy of equivocation, switching from one meaning to another of an ambiguous term within the course of a single argument. Formally, it’s analogous to this:

Clinton built the Erie Canal.
Clinton = Hillary Rodham’s husband.
Therefore, Hillary Rodham’s husband built the Erie Canal.

In the first premiss, “Clinton” refers to DeWitt Clinton, whereas in the second, it refers to Bill Clinton.

A big push in logical research at the moment is the attempt to extend the predicate
calculus to incorporate Frege’s insights. Unfortunately, the path is rugged and progress is slow. We won’t be able to go into the issues here.*