Schemas

A schema is a sentence or phrase with some words (phrases, sentences) taken out and replaced by schematic symbols—usually either Roman or Greek letters, or ellipses, or a blank. A schema comes paired with a rule or side condition, specifying how new sentences or phrases may be generated by replacing the schematic symbols with words (phrases, sentences). Often the rule is left implicit.

Schemas are often used to give the axioms of a logical or mathematical theory. For example, in a logic textbook one might read:

Axiom 1: \( \phi \supset (\phi \supset \psi) \)

In fact, ‘(\( \phi \supset (\phi \supset \psi) \))’ is not an axiom. Rather, it is an axiom schema which together with a rule (replace ‘\( \phi \)’ by any sentence of the logical language being axiomatized, ditto ‘\( \psi \)’) generates infinitely many axioms. In other words:

Axioms of the first kind: Every result of uniformly replacing every occurrence of ‘\( \phi \)’ with a sentence and every occurrence of ‘\( \psi \)’ with a sentence in the following schema is an axiom:

\( (\phi \supset (\phi \supset \psi)) \)

A famous schema is Tarski’s Schema T: \( x \) is a true sentence if and only if \( p \).

Often schemas are implicit in philosophical writing. For instance, one might read:

If \( S \) knows that \( p \), \( S \) believes that \( p \)

with no accompanying explanation of ‘\( S \)’ and ‘\( p \)’. What is probably intended is something like:

Every result of replacing, in the following sentence, every occurrence of ‘\( S \)’ with the name of a person and every occurrence of ‘\( p \)’ with a declarative sentence of English is a true sentence:

\( (*) \quad \text{If } S \text{ knows that } p, S \text{ believes that } p \)

Corners could be used to exactly the same effect:
For all names \( a \) and declarative sentences \( b \), \([\text{If } a \text{ knows that } b, \ a \text{ believes that } b\])\) is a true sentence.

'\( a \)' above is a variable ranging over expressions, but '\( p \)' (as used in the schema (*) is not. Thus (*) plus accompanying rule is to be sharply distinguished from:

\[ \text{(**) \quad \text{For all } S, \ p, \text{ if } S \text{ knows that } p, \ S \text{ believes that } p.} \]

Here '\( S \)' and '\( p \)' are used as variables. The quantification into sentence position is a kind of "higher-order" quantification (as opposed to the "first order" quantification into subject position). Just how to interpret such higher-order quantification is controversial. In contrast, the schema is unproblematic.

Incidentally, in this particular case controversies about higher-order quantification can be sidestepped while retaining '\( p \)' as a variable, by replacing (**)) with (***):

\[ \text{(***) \quad \text{For all } S, \ p, \text{ if } S \text{ knows } p, \ S \text{ believes } \ p} \]

Here '\( p \)' is a first-order variable ranging over propositions.

For a bit more, see [http://plato.stanford.edu/entries/schema/](http://plato.stanford.edu/entries/schema/) and the first two pages from Quine's "The Variable" (from his collection The Ways of Paradox).