Problem Set #6

PROBLEM SET #6

Read the two page handout giving an overview of the system

Using the sample calculations as a model, calculate the truth-conditions (starting with an arbitrary filler $a$ and an arbitrary world $w$) for the following sentence:

\[ \text{Hannibal is a dog who saw Shelby.} \]

For each step of your calculation, give an annotation of what justifies the step (lexicon entry, the FA, PA, PM principles, the definition of the $\lambda$-notation).
1. The Lambda Notation for Functions

(1) Read “[\lambda \alpha . \beta]” as either (i) or (ii), whichever makes sense.
   (i) “the function which maps every \alpha to \beta”
   (ii) “the function which maps every \alpha to 1, if \beta, and to 0 otherwise”

2. Lexicon

Some words:

(2) For any world \(w\) and (filler) individual \(a\),
   a. \([\text{Shelby}]^{w,a} = \text{Shelby}\).
   b. \([\text{Hannibal}]^{w,a} = \text{Hannibal}\).
   c. \([\text{barks}]^{w,a} = \lambda x. \text{barks in } w\).
   d. \([\text{dog}]^{w,a} = \lambda x. \text{x is a dog in } w\).
   e. \([\text{smart}]^{w,a} = \lambda x. \text{x is smart in } w\).
   f. \([\text{saw}]^{w,a} = \lambda x. \lambda y. \text{y saw x in } w\).

The special rule for traces:

(3) For any world \(w\) and (filler) individual \(a\),
\([t]^{w,a} = a\).

3. Functional Application

(4) Functional Application (FA)
   For any world \(w\) and (filler) individual \(a\), if \(\alpha\) is a branching node, \(\{\beta, \gamma\}\) the set of \(\alpha\)’s daughters, and \([\beta]^{w,a}\) is a function whose domain contains \([\gamma]^{w,a}\), then \([\alpha]^{w,a} = [\beta]^{w,a}([\gamma]^{w,a})\).

4. Predicate Abstraction

(5) Predicate Abstraction (PA)
   For any world \(w\) and (filler) individual \(a\), if \(\alpha\) is a branching node whose daughters are a relative pronoun and \(\beta\), then \([\alpha]^{w,a} = \lambda x. [\beta]^{w,x}\).
5. Predicate Modification

(6) Predicate Modification (PM)
For any world $w$ and (filler) individual $a$, if $\alpha$ is a branching node, $\{\beta, \gamma\}$ the set of $\alpha$'s daughters, and if $[\beta]^{w,a}$ and $[\gamma]^{w,a}$ are both functions from individuals to truth-values (one-place predicates), then $[\alpha]^{w,a} = \lambda x. [\beta]^{w,a}(x) = [\gamma]^{w,a}(x) = 1$.

6. Two Sample Calculations

Pick an arbitrary filler, say $a$ and an arbitrary world $w$.

(7) $[[\text{Hannibal is a smart dog}]]^{w,a}$
$= [[\text{Hannibal} (\text{smart dog})]]^{w,a}$
$= [\text{smart dog}]^{w,a}([[\text{Hannibal}]]^{w,a})$
$= [\text{smart dog}]^{w,a} (\text{Hannibal})$
$= [\lambda x. [\text{smart}]^{w,a}(x) = [\text{dog}]^{w,a}(x) = 1] (\text{Hannibal})$
$= 1$ iff $[\lambda x. x \text{ is smart in } w] (\text{Hannibal}) = [\lambda x. x \text{ is a dog in } w] (\text{Hannibal}) = 1$
iff Hannibal is smart in $w$ and Hannibal is a dog in $w$.

(8) $[[\text{Hannibal is who Shelby saw } t]]^{w,a}$
$= [[\text{who Shelby saw } t]]^{w,a}([[\text{Hannibal}]]^{w,a})$
$= [[\text{who Shelby saw } t]]^{w,a} (\text{Hannibal})$
$= [\lambda x. [\text{Shelby saw } t]^{w,x} (\text{Hannibal})$
$= [[\text{Shelby saw } t]]^{w,\text{Hannibal}}$
$= [[\text{saw}]^{w,\text{Hannibal}} (t)]^{w,\text{Hannibal}} ([[\text{Shelby}]]^{w,\text{Hannibal}})$
$= [[\text{Hannah saw } t \text{ in } w] (\text{Hannibal})] (\text{Shelby})$
$= [\lambda y. [\text{you saw } x \text{ in } w] (\text{Hannibal})] (\text{Shelby})$
$= [\lambda y. [\text{you saw Hannibal in } w]] (\text{Shelby})$
$= 1$ iff Shelby saw Hannibal in $w$.

7. Problem Set #6

Using the sample calculations as a model, calculate the truth-conditions (starting with an arbitrary filler $a$ and an arbitrary world $w$) for the following sentence:

$\text{Hannibal is a dog who } t \text{ saw Shelby}$.

For each step of your calculation, give an annotation of what justifies the step (lexicon entry, the FA, PA, PM principles, the definition of the $\lambda$-notation).