(1) The S-Exhaustivity Generalization (predicted by Sauerland’s Theory): utterance of a sentence, S, as a default, licenses the inference that (the speaker believes that) every sentence is false if it is Sauerland-Excludable given S and Alt(S).

p is Sauerland-Excludable given S and C if p∈C, p is stronger than S and ¬∃q∈C [(q is stronger than S) and (S∧¬p entails q)].

Homework:

Prove that the Sauerland-Exhaustivity Generalization is indeed predicted by Sauerland’s theory.

Solution to question 1:

Let p be Sauerland-Excludable given S and Alt(S). We need to prove that

\( B_s(S) \land \bigcap PI \land B_s(\neg p) \) is not contradictory

Assume otherwise: (and try to derive a contradiction)

(a) \( B_s(S) \land \bigcap PI \land B_s(\neg p) \) is contradictory.

We conclude:

(b) \( B_s(S) \land B_s(\neg p) \) entails \( \neg \bigcap PI \)

(c) \( B_s(S) \land B_s(\neg p) \) entails \( \bigcup \neg PI \) (De Morgan)

(d) \( B_s(S) \land B_s(\neg p) \) entails \( \bigcup \{ B_s(q): q \in \text{Alt}(S) \text{ and } q \text{ stronger than } s \} \)

Let \( w^0 \) be a world in which s believes nothing but S and \( \neg p \) (and their logical consequences).

(e) \( w^0 \) satisfies \( \bigcup \{ B_s(q): q \in \text{Alt}(S) \text{ and } q \text{ stronger than } s \}. \) (given the entailment in (d))

For a world to satisfy a disjunction, it must satisfy one of the disjuncts.

So

(f) there must be a \( q_i \in \text{Alt}(S) \), stronger than s such that \( q' \) is a logical consequence of S and \( \neg p \).

Hence,

(g) p is not Sauerland-Excludable.

Note: it is easier to prove the other direction, i.e. \( \forall p \in \text{ALT}(S)(B_s(\neg p) \) is a Secondary Implicature of S by Sauerland’s algorithm \( \rightarrow p \) is Sauerland Excludable given S and ALT(S)).