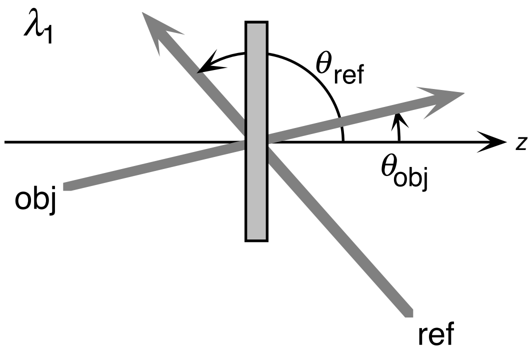
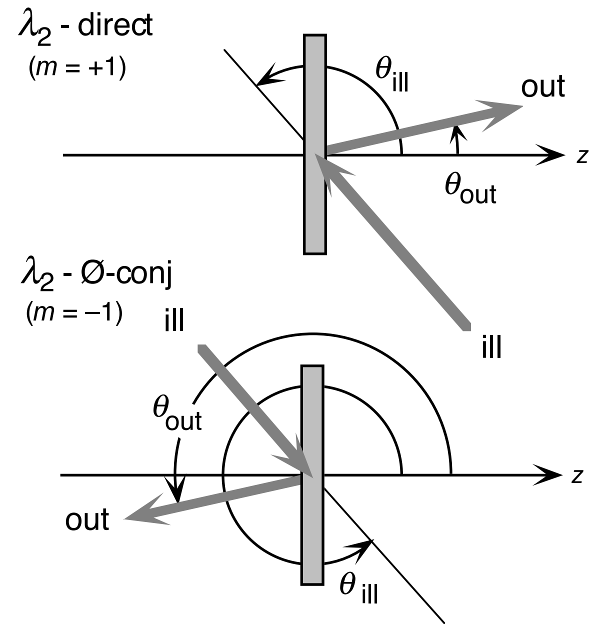
REFLECTION RAY-TRACING:

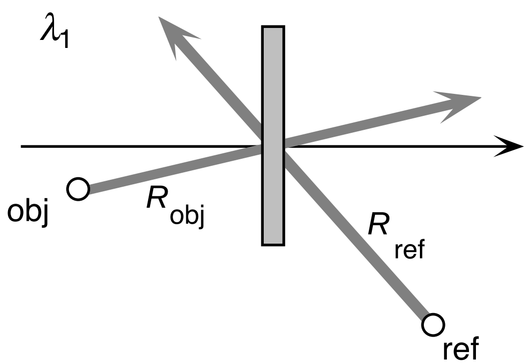
reference & illumination <u>angles</u> are measured “the long way around” and the object beam is in the $+z$ direction

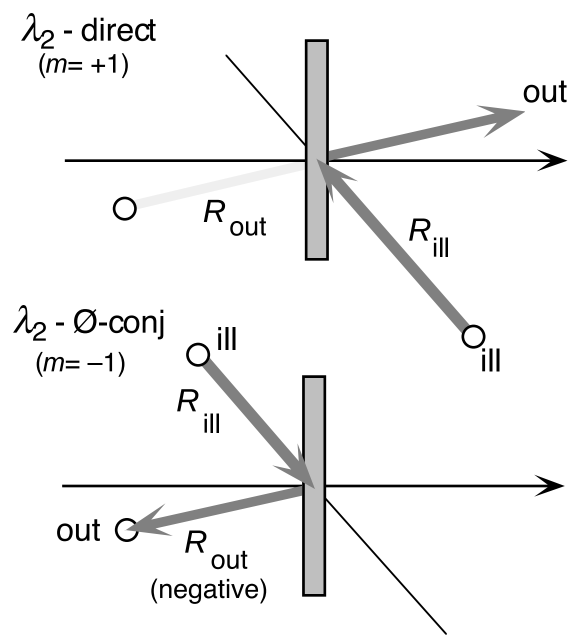




$m = +1$ for “direct” reconstruction
(illum & ref from **same** side)
-1 for “phase conjugation”
(illum & ref from **opposite** sides)

<u>distances</u> = radii of curvature (negative => real image)





HORIZONTAL FOCUS (<u>out</u> of the plane of the page)-

$$\frac{\dfrac{1}{R_{out}} - \dfrac{1}{R_{ill}}}{\lambda_2} = m\,\frac{\dfrac{1}{R_{obj}} - \dfrac{1}{R_{ref}}}{\lambda_1}$$

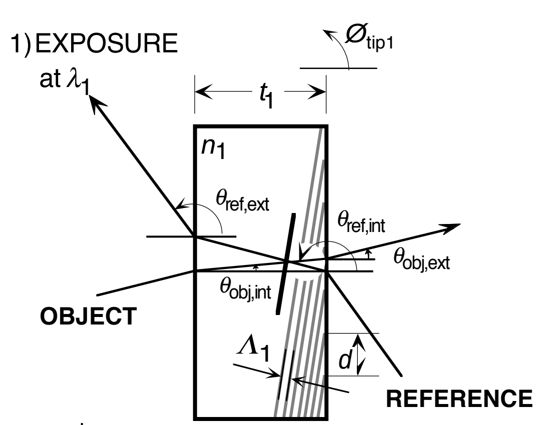
VERTICAL FOCUS (<u>in</u> of the plane of the page)-

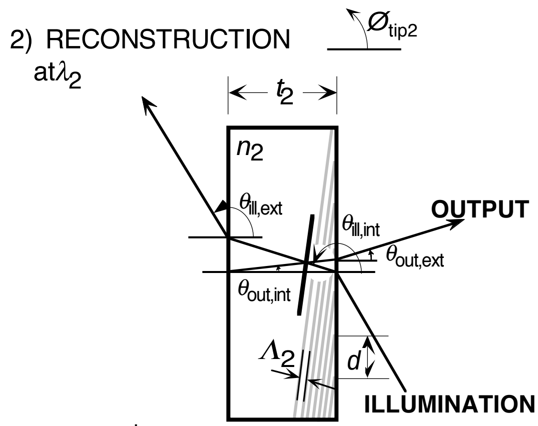
$$\frac{\dfrac{\cos^2\theta_{out}}{R_{out}} - \dfrac{\cos^2\theta_{ill}}{R_{ill}}}{\lambda_2} = m\,\frac{\dfrac{\cos^2\theta_{obj}}{R_{obj}} - \dfrac{\cos^2\theta_{ref}}{R_{ref}}}{\lambda_1}$$

<u>All</u> angles on this page are the usual **external** angles.
The usual time that internal angles (the θ') are used is in the “fringe-tip and -separation” calculations, and the “z-equation” for the allowed angles (if you use that approach).

The angles and wavelengths are first determined by those calculations, and are then plugged in to these focus equations to solve the imaging questions.

Off-Axis Reflection Holography
(direct, forward, m=+1 reconstruction)





recall: Snell's Law: $\sin\theta_{xxx,\mathrm{ext}} = n_i \cdot \sin\theta_{xxx,\mathrm{int}}$ also: $n_{\mathrm{ext}} \cdot \lambda_{\mathrm{ext}} = n_{\mathrm{int}} \cdot \lambda_{\mathrm{int}}$

tilted - stacked - mirror representation:

$$t_1 \cdot \tan\Phi_{\mathrm{tip1}} = t_2 \cdot \tan\Phi_{\mathrm{tip2}}$$

$$\Phi_{\mathrm{tip1}} = \frac{\theta_{\mathrm{obj,int}} + \theta_{\mathrm{ref,int}}}{2} \quad , \quad \Phi_{\mathrm{tip2}} = \frac{\theta_{\mathrm{out,int}} + \theta_{\mathrm{ill,int}}}{2}$$

$$\frac{t_1}{\Lambda_1}\sin\Phi_{\mathrm{tip1}} = \frac{t_2}{\Lambda_2}\sin\Phi_{\mathrm{tip2}}$$

$$\frac{1}{\Lambda_1} = \frac{2}{\lambda_{1,\mathrm{int}}}\cos\left(90^\circ + \frac{\theta_{\mathrm{obj,int}} - \theta_{\mathrm{ref,int}}}{2}\right) \quad , \quad \frac{1}{\Lambda_2} = \frac{2}{\lambda_{2,\mathrm{int}}}\cos\left(90^\circ + \frac{\theta_{\mathrm{out,int}} - \theta_{\mathrm{ill,int}}}{2}\right)$$

x-, z - grating representation (all m):

$$\frac{\sin\theta_{\mathrm{obj,ext}} - \sin\theta_{\mathrm{ref,ext}}}{\lambda_{1,\mathrm{ext}}} = \frac{1}{d} = m\frac{\sin\theta_{\mathrm{out,ext}} - \sin\theta_{\mathrm{ill,ext}}}{\lambda_{2,\mathrm{ext}}}$$ $\Leftarrow$ means that $1/R$ and $\cos^2\theta/R$ still work!

$$n_1 \cdot t_1 \frac{\cos\theta_{\mathrm{obj,int}} - \cos\theta_{\mathrm{ref,int}}}{\lambda_{1,\mathrm{ext}}} = n_2 \cdot t_2 \cdot m\frac{\cos\theta_{\mathrm{out,int}} - \cos\theta_{\mathrm{ill,int}}}{\lambda_{2,\mathrm{ext}}}$$ (± 1, Goodman - Heisenberg Uncertainty)

Special Case: On-Axis Reflection "Denisyuk" Holography
(direct, forward, m=+1 reconstruction)

$$\theta_{\mathrm{ref,ext}} = 180^\circ - \theta_{\mathrm{obj,ext}}\ , \quad \text{so } \Phi_{\mathrm{tip1}} = \Phi_{\mathrm{tip2}} = 90^\circ \ \text{(conformal fringes)}$$

so that: $\theta_{\mathrm{out,ext}} = 180^\circ - \theta_{\mathrm{ill,ext}}$ (mirror reflection)

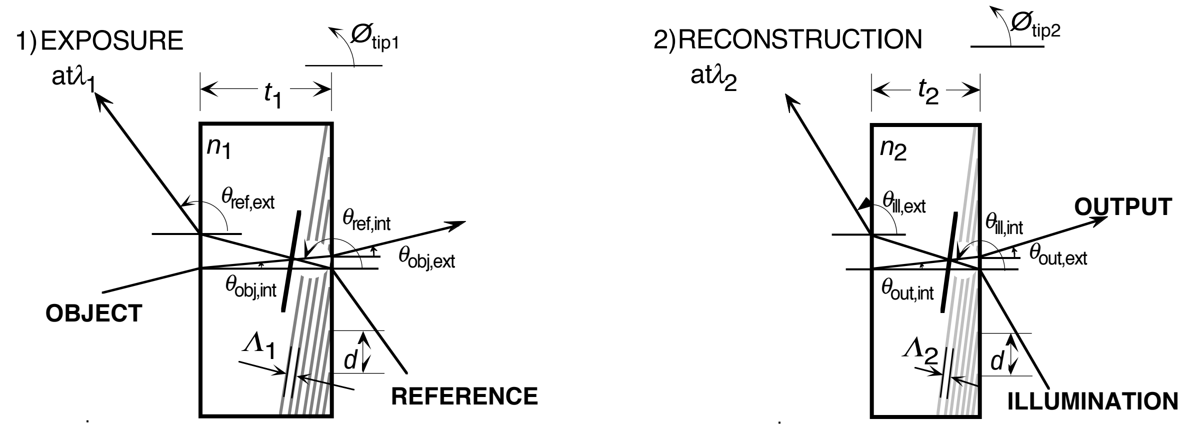
$$\frac{1}{\Lambda_1} = \frac{2 \cdot n_1}{\lambda_{1,\mathrm{ext}}}\cos\left(\theta_{\mathrm{obj,int}}\right)$$

$$\frac{t_1}{\Lambda_1} = \frac{t_2}{\Lambda_2}$$

$$\frac{1}{\Lambda_2} = \frac{2 \cdot n_2}{\lambda_{2,\mathrm{ext}}}\cos\left(\theta_{\mathrm{out,int}}\right) ,$$

or pulling it together: $n_1 \cdot t_1 \dfrac{\cos\theta_{\mathrm{obj,int}}}{\lambda_{1,\mathrm{ext}}} = n_2 \cdot t_2 \dfrac{\cos\theta_{\mathrm{out,int}}}{\lambda_{2,\mathrm{ext}}}$ (± 1, GHU)

RSHRINK: definitions of variables



Reflection Gratings, forward reconstruction (m=1): RSHRINK **pseudo-rules** for *TK-Solver+*:

$\angle$refl = $-\angle$incidence

1) $\varnothing_{tip1} = (\theta_{obj,int} + \theta_{ref,int})/2$ — "fringe tip angle during exposure"

2) $\varnothing_{tip2} = (\theta_{out,int} + \theta_{ill,int})/2$ — "fringe tip angle during reconstruction"

3) $t_2 = (1 - \text{shrink}/100) \cdot t_1$ — "simple shrinkage definition"

4) $t_1 \cdot \tan(\varnothing_{tip1}) = t_2 \cdot \tan(\varnothing_{tip2})$ — "fringe *x*-span constancy"

cosθ x λ "blue shift"

5) $diff_1 = (\theta_{ref,int} - \theta_{obj,int})/2$ — "*diff*s are handy variables"

6) $diff_2 = (\theta_{ill,int} - \theta_{out,int})/2$

7) $(1/\Lambda_1) = (2/\lambda_{1int}) \cdot \cos(90° + diff_1)$ — "fringe separation during exposure"

8) $(1/\Lambda_2) = (2/\lambda_{2int}) \cdot \cos(90° + diff_2)$ — "fringe separation during reconstruction"

9) $d \cdot \cos(\varnothing_{tip1}) = \Lambda_1$ — "change of fringe separation"

10) $d \cdot \cos(\varnothing_{tip2}) = \Lambda_2$

11) $t_1 \cdot \sin(\varnothing_{tip1})/\Lambda_1 = t_2 \cdot \sin(\varnothing_{tip1})/\Lambda_2$

internal vs. external

12) $\sin(\theta_{obj,ext}) = n_1 \cdot \sin(\theta_{obj,int})$ — "Snell's Law sequence"

13) $\sin(180° - \theta_{ref,ext}) = n_1 \cdot \sin(180° - \theta_{ref,int})$ — "in case arcsin is limited to quads I & IV"

14) $\sin(180° - \theta_{ill,ext}) = n_2 \cdot \sin(180° - \theta_{ill,int})$

15) $\sin(\theta_{out,ext}) = n_2 \cdot \sin(\theta_{out,int})$

16) $\lambda_{1ext} = n_1 \cdot \lambda_{1int}$ — "int vs. ext for wavelengths"

17) $\lambda_{2ext} = n_2 \cdot \lambda_{2int}$

misc. cleanup

18) $\lambda_{1ext}/d = \sin(180° - \theta_{ref,ext}) - \sin(\theta_{obj,ext})$ — "interference equation, *x*-component"

19) $\lambda_{2ext}/d = \sin(180° - \theta_{ill,ext}) - \sin(\theta_{out,ext})$ — "diffraction equation, *x*-component"

20) $(\cos(\theta_{ref,int}) - \cos(\theta_{obj,int})) \cdot (1/\lambda_{1int}) \cdot (t1/t2) =$
$(\cos(\theta_{ill,int}) - \cos(\theta_{out,int})) \cdot (1/\lambda_{2int})$ — "*z*-component match"

20) $\theta_{ref,int} = \varnothing_{tip1} + diff_1$ — "restatements, if needed"

21) $\theta_{ill,int} = \varnothing_{tip2} + diff_2$