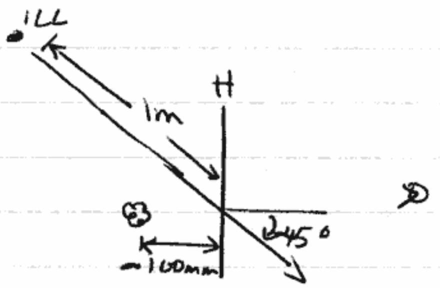
MAS 450/854 Problem Set #3 Sol'ns

① a+b



need to find angle and position (well, R) of reference + object when hologram is made in 633nm light to compensate for change in wavelength at illumination (530nm)

angle:

solve for $\sin\theta_{Ref}$, since $\sin\theta_{OBJ}$ is given as 0.

$$\sin\theta_{OUT} = m\frac{\lambda_2}{\lambda_1}(\sin\theta_{OBJ} - \sin\theta_{Ref}) + \sin\theta_{ILL}$$

$$0 = \overset{1}{\cancel{m}}\frac{530}{633}(0 - \sin\theta_{Ref}) + \sin(45)$$

$$\sin\theta_{Ref} = 1/\left(\frac{530}{633}\right)\cdot\sin(45)$$

$$\theta_{Ref} = -58^\circ$$

now, find location of the reference and object at exposure. Unlike the angle of the reference source, the R equation isn't as strongly constrained by the customer - both R_{ref} and R_{OBJ} can be varied based on the customer's strict requirements. Solving the horizontal focus equation:

$$\frac{1}{R_{OUT}} = m\frac{\lambda_2}{\lambda_1}\left(\frac{1}{R_{OBJ}} - \frac{1}{R_{ref}}\right) + \frac{1}{R_{ILL}}$$ for unknowns:

$$\frac{1}{R_{OBJ}} - \frac{1}{R_{Ref}} = \frac{1}{\underset{1}{\cancel{m}}}\frac{\overset{633}{\cancel{\lambda_1}}}{\underset{530}{\cancel{\lambda_2}}}\left(\overset{100}{\cancel{\boxed{\frac{1}{R_{OUT}}}}} - \overset{1000}{\boxed{\frac{1}{R_{ILL}}}}\right)$$

$$\frac{1}{R_{OBJ}} - \frac{1}{R_{Ref}} = 1\,\frac{633}{530}\left(\frac{1}{100} - \frac{1}{1000}\right)$$

So, this equation allows some flexibility in choosing values, based on customer needs or physical constraints.

The customer is, however, concerned at some level about magnification, so one approach is to minimize it (closest to 1):

$$MAG_{lat} = m \frac{\lambda_{ILL}}{\lambda_{exp}} \frac{R_{OUT}}{R_{OBJ}} \Rightarrow$$

$$1 = (1)\, \frac{530}{633} \cdot \frac{100}{R_{OBJ}}$$

$$R_{OBJ} \cong 84\,mm \qquad (83.73)$$

substituting symbolically:

(from above): $$\frac{1}{R_{OUT}} \cdot \frac{\lambda_{exp}}{\lambda_{ILL}} \frac{1}{m} = \frac{1}{R_{OBJ}}$$

(from last page): $$\frac{1}{R_{OBJ}} - \frac{1}{R_{REF}} = \frac{1}{m} \frac{\lambda_1}{\lambda_2} \left(\frac{1}{R_{OUT}} - \frac{1}{R_{ILL}} \right)$$

$$\frac{1}{R_{REF}} = \frac{1}{m} \frac{\lambda_1}{\lambda_2} \left(\frac{1}{R_{ILL}} - \frac{1}{R_{OUT}} \right) + \frac{1}{R_{OBJ}}$$

$$\frac{1}{R_{REF}} = \frac{1}{m} \frac{\lambda_1}{\lambda_2} \left(\frac{1}{R_{ILL}} - \frac{1}{R_{OUT}} \right) + \frac{1}{m} \frac{\lambda_1}{\lambda_2} \frac{1}{R_{OUT}}$$

$$= \frac{1}{m} \frac{\lambda_1}{\lambda_2} \left(\frac{1}{R_{ILL}} \cancel{- \frac{1}{R_{OUT}} + \frac{1}{R_{OUT}}}^{\,0} \right)$$

$$\frac{1}{R_{ref}} = \frac{1}{m} \frac{\lambda_1}{\lambda_2} \left(\frac{1}{R_{ILL}} \right)$$

$$R_{ref} = m \frac{\overset{530}{\cancel{\lambda_2}}}{\underset{633}{\cancel{\lambda_1}}} R_{ILL}$$

$$\boxed{R_{ref} = 840\,mm}$$

Ⓒ → MAG lat Horiz focus set to be 1, see mlong next page

since the problem set did not offer a "hard constraint" for Rref, other reasonable answers will be accepted. Reasonable: not collimated or negative ("you don't have a large lens") not very long or very short.

Big picture – we move the object and reference in, and steepen the reference angle, so that illumination in a shorter wavelength will push the image back.

① d. horizontal lines of object $\Rightarrow$ vert. focus $\Rightarrow \frac{\cos^2\theta}{R}$

$$\frac{\cos^2\theta_{OUT}}{R_{OUT}} = m\,\frac{\lambda_2}{\lambda_1}\left(\frac{\cos^2\theta_{OBJ}}{R_{OBJ}} - \frac{\cos^2\theta_{ref}}{R_{REF}}\right) + \frac{\cos^2\theta_{ILL}}{R_{ILL}}$$

$$\frac{1}{R_{OUT}} = \overset{1}{\cancel{m}}\,\frac{530}{633}\left(\frac{1}{R_{OBJ}} - \frac{\cos^2\theta_{Ref}}{R_{Ref}}\right) + \frac{\cos^2\theta_{ILL}}{R_{ILL}}$$

$$\frac{1}{R_{OUT}} = \frac{530}{633}\left(\frac{1}{84} - \frac{\cos^2(-58)}{840}\right) + \frac{\cos^2(-45)}{1000}$$

$$R_{OUT} \cong 98mm$$

ⓔ lateral magnification vertical focus:

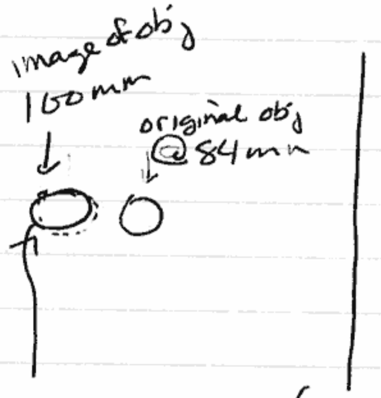
$$MAG_{lat} = m\,\frac{530}{633}\,\frac{98}{84} = \boxed{0.98}$$

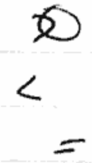
not required on P.S. $\rightarrow$ $\left(MAG_{long} = \frac{530}{633}\left(\frac{98}{84}\right)^2 = 1.14\right)$

ⓒ MAG$_{long}$ vertical lines $\Rightarrow$ horizontal focus.

$$MAG_{long}:\ m\,\frac{\lambda_{ILL}}{\lambda_{exp}}\left(\frac{R_{OUT}^2}{R_{OBJ}^2}\right)$$

$$(1)\frac{530}{633}\left(\frac{100^2}{84^2}\right) = \boxed{1.2}$$



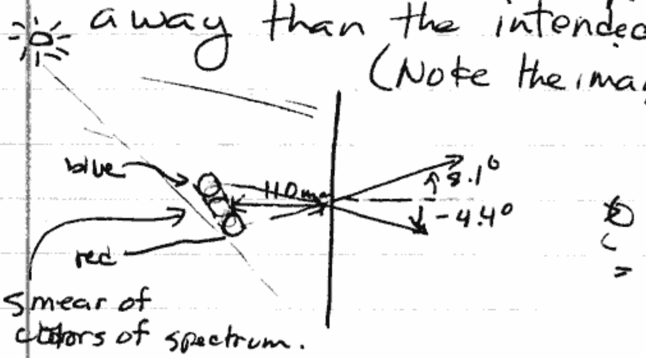


slightly enlongated (1.2:1)
some astigmatism, vertical focus slightly in front (2mm)
and slightly smaller.

② Sunlight illumination:

The object will appear behind the plate, but the color range of the sunlight will smear the image vertically, with the red light image appearing on the bottom and slightly closer to the plate than the blue light image (on the top and further away from the plate). Since the sun is "collimated" further away than 1000mm, the object will also appear somewhat further away than the intended 1000 mm in 530nm wavelength.

(Note the image is a continuous smear)



$$\frac{1}{R_{OUT}} = m\frac{\lambda_2}{633}\left(\frac{1}{R_{OBJ}} - \frac{1}{R_{ref}}\right) + \frac{1}{R_{ILL}}$$

whatever the values for R_{OBJ} and R_{REF} you previously found, they must satisfy: $\left(\frac{1}{R_{OBJ}} - \frac{1}{R_{REF}}\right) = \frac{633}{530}\left(\frac{1}{100} - \frac{1}{1000}\right)$

substitute:

$$\frac{1}{R_{OUT}} = m\frac{\lambda_2}{\cancel{633}}\left(\frac{\cancel{633}}{530}\left(\frac{1}{100} - \frac{1}{1000}\right)\right) + \overset{0}{\cancel{\frac{1}{\boxed{R_{ILL}}}}} \quad R_{ILL} \rightarrow \infty$$

$$\frac{1}{R_{OUT}} = m\frac{\lambda_2}{530}\left(\frac{1}{100} - \frac{1}{1000}\right)$$

$$\sin\theta_{OUT} = \frac{\lambda_2}{633}\left(0 - \sin(-58)\right) + \sin(-45)$$

$\lambda_2 = 470$nm: $R_{OUT} = 125$mm

$\lambda_2 = 530$nm: $R_{OUT} = 110$mm

$\lambda_2 = 633$nm: $R_{OUT} = 93$ mm

$\lambda_2 = 470$nm: $\theta_{OUT} = -4.4°$

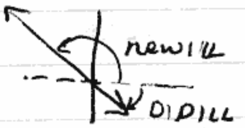
$\lambda_2 = 530$nm $\theta_{OUT} = 0°$

$\lambda_2 = 633$nm $= 8.1°$

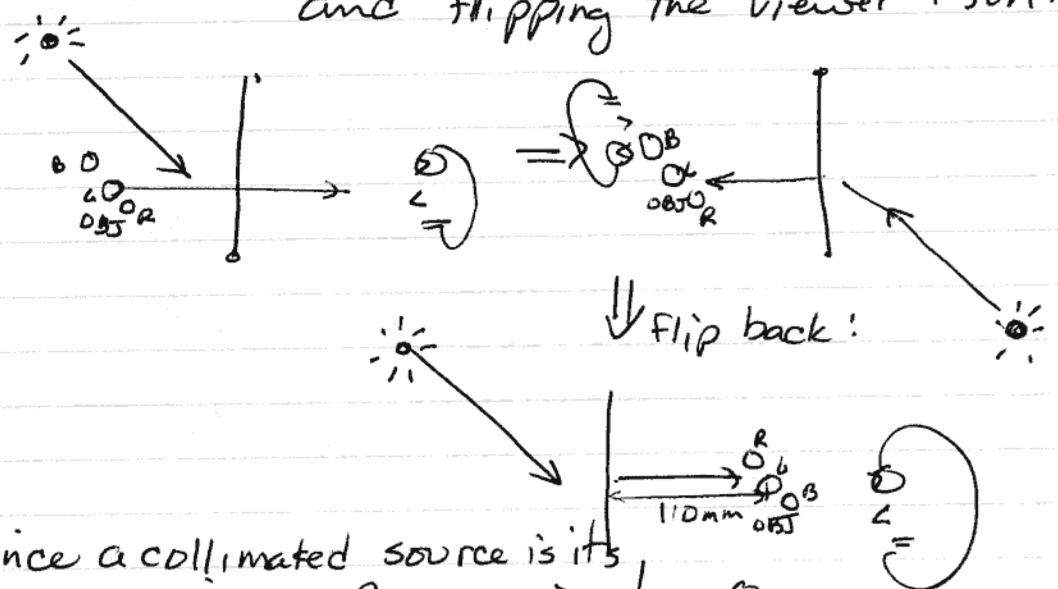
2c: "illuminated by the sun from exactly the opposite angle as before"

⇒ collimated and conjugate illumination

$$\theta_{newILL} = (180^\circ - \theta_{OLDILL})$$

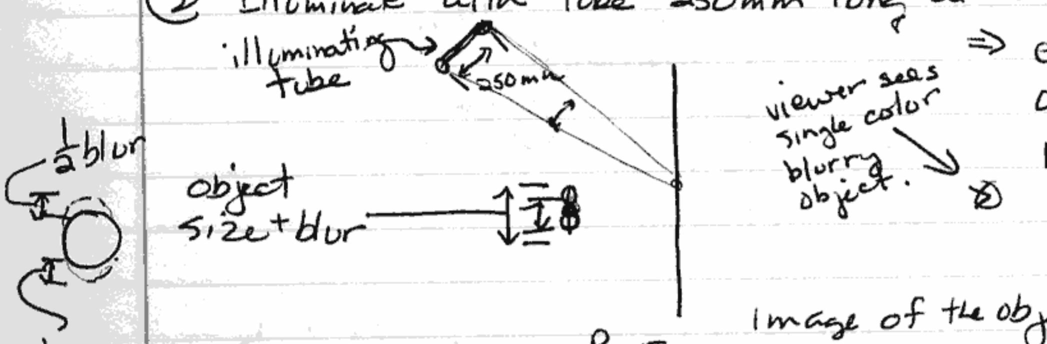


we can recast the problem by keeping the plate fixed and flipping the viewer + sun:



Since a collimated source is its own conjugate, $R_{ILL} = \infty \Rightarrow \frac{1}{R_{ILL}} = 0$, and the $m=-1$ order images ~~is~~ are the same distance from the plate as you found in part (b). The sign of R is now negative, which means the image is a real image (converging) in front of plate.

③ Illuminate with tube 250mm long at 1000m:



⇒ every point in the object becomes an image of the tube, or, each point on the tube forms one image of the object. A rough estimate of blur size is $250mm \cdot \frac{R_{OUT}}{R_{ILL}}$ or 25mm, plus the object size. You could be more exact and compute $\sin\theta$ for top + bottom of tube ⇒ 18mm