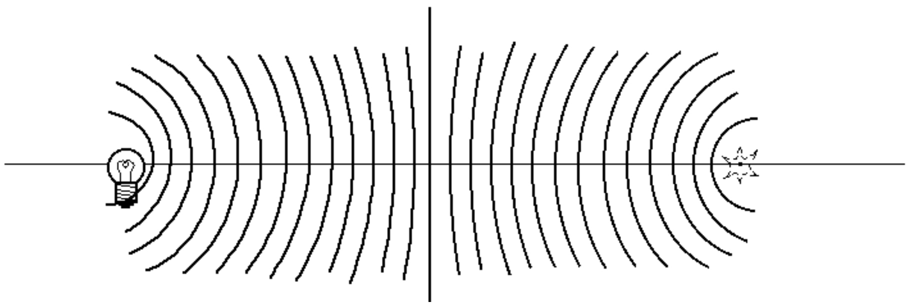
Ch. 16: In-line “Denisyuk” Reflection Holograms

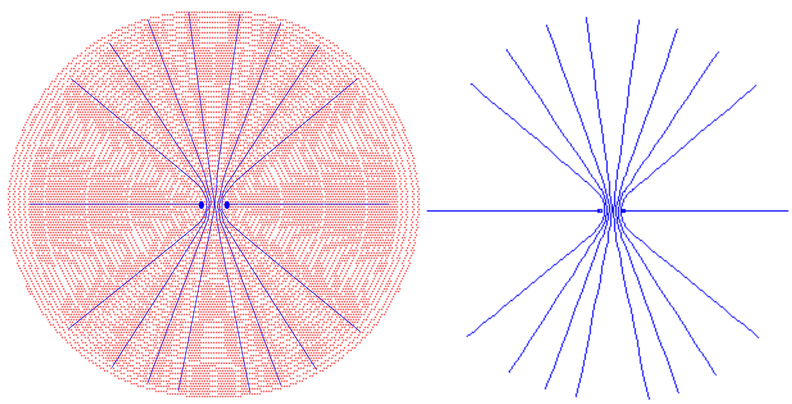
(preliminary draft, April 2003 — bug reports appreciated)

We have been thinking of transmission holograms as producing images by means of an array of many overlapping negative lenses of various locations and focal length. Each elemental lens forms a virtual image behind the hologram plane. With each negative lens there is an associated positive lens that forms an image in front of the hologram too, and unlike glass lenses these diffraction lenses form images in different locations for different colors. But there is another way of forming a virtual image of a point illumination source: by reflection from a convex (outwardly curving) mirror surface!

Consider a point source of light, and a location at which we would like to form an image. There are an infinite number of locations for mirrors of various curvatures (positive = convex, none = planar, negative = concave) that will do the job. For example, a flat mirror forms a virtual image at a distance behind the mirror equal to the distance to the source in front (recall that your image in a flat mirror is as far behind the mirror as you are in front). Any of these mirrors will produce the image we seek, or a stack of barely-reflecting mirrors if the variation of curvature with location is correct. We are going to think of a reflection hologram as a slice or sample though a stack like this one!



But how to fabricate such a stack of mirrors? The solution to this puzzle was provided by the Russian physicist Yurii Nikolaevich Denisyuk in 1958 (published in Russian in 1962[1] and in English in 1963[2]). Recall the interference pattern formed between two point sources, such as we considered in Chapter 4. One of the surfaces of constructive interference, or reinforcement, is a flat plane halfway between the two sources. To the right of that mid-point, the surfaces curve toward the right-hand source, as spheres near the axis but stretching out to become hyperpoloids of revolution. To the left, they curve in the opposite direction.



Denisyuk’s insight was to use these interference patterns to expose a very-high-resolution emulsion, and to process the patterns to produce reflecting “fringes” that would be nested with exactly the proper shape to serve as the mirrors mentioned above. Instead of using “Lippmann emulsions,” he used “Kirillov emulsions,” which had been invented in the Soviet Union a few years before, and are even finer-grained than the French/Belgian products. Note that Denisyuk’s work preceded the invention of the laser by about four years! He used a special high-pressure mercury lamp which produced light that was weak but highly coherent—it was probably a superradiant source, in today’s terms. Denisyuk’s ideas about capturing the shape of optical wavefronts by interference, and reconstructing them by diffraction, were met with deep suspicion by the Russian Academy of Sciences, and his work was suppressed until the later work by Leith and Upatnieks drew international attention. Then Peter Kaptiza required that his critics write letters of support for the Lenin Prize (their highest scientific recognition), which Denisyuk received in 1969. From then onwards, holography was a prominent feature of the Soviet Union’s scientific profile, along with space technology, nuclear power, and high-power lasers. We refer to such holograms as “Denisyuk,” or “volume reflection” or “volume dielectric” holograms (especially to distinguish them from the “reflective rainbow” holograms mentioned previously).

The source of Denisyuk’s remarkable idea was a boyhood reading of the story “Star Ships” by the noted science fiction author J.A. Efremov (also translated into English). In it he described travelers who found a multilayered metal disk. When the sun shined on it, 3D images of humanoid faces appeared. Denisyuk took on this scientific challenge, and soon realized that it was similar to the “interference color” images of Gabriel Lippmann. It took a while, but Denisyuk eventually realized his boyhood dream.

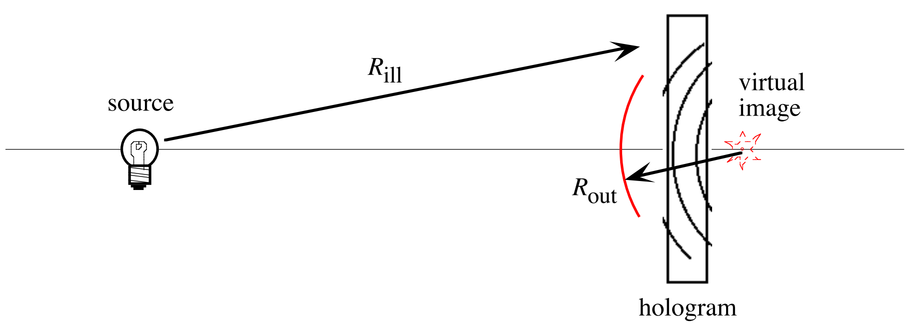
Making a Denisyuk hologram:

The “classical” or “single-beam” Denisyuk technique simply shines a diverging laser beam through a holographic plate, which is so finely-grained that it absorbs very little of the light, and onto the subject of the hologram. The light reflects back from the subject to the plate, where it overlaps the incoming light to produce the desired interference pattern. Of course the subject must closer to the plate than one-half the laser’s coherence length, but otherwise the technique is very simple and direct, and can produce results of very high quality. It is well-suited to very large holograms, because no supplemental optics are required and the system is readily engineered to be resistant to vibration.

But a more important property of reflection holograms is that they can be viewed with white light from concentrated sources. That is because they reflect only a narrow spectrum, usually centered at the same wavelength as that of the exposing laser. This is because the stacked mirrors are uniformly spaced by half the wavelength of the reflected light, just as the interference fringes that produced them were. Light first reflects from the first curved mirror surface, but most of it passes on to the deeper layers. The reflection from the second layer comes back out after a delay of one full wavelength. All the following reflections are delayed by one more wavelength each. If there are enough such reflections of roughly equal strength, then only one wavelength is strongly reflected (the one for which all the reflections emerge in phase), and the strength drops to one-half its maximum if the wavelength varies by one-over-*M* of its central value (where *M* is the number of reflections that come back).

The optics of in-line reflection holograms: distances

The distance-determining optics of the reflecting diffractive mirrors are the same as for transmissive diffractive lenses, and we will cite the relevant equations without proof. Unlike normal mirrors, they have a strong wavelength dependence, but otherwise they have many of the same properties as conventional mirrors.

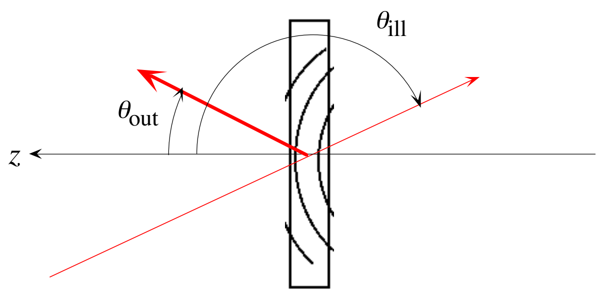


$$\frac{1}{R_{\text{out}}}-\frac{1}{R_{\text{ill}}}=m\frac{\lambda_2}{\lambda_1}\left(\frac{1}{R_{\text{obj}}}-\frac{1}{R_{\text{ref}}}\right)=\frac{1}{R_{\text{effective-mirror}}} \tag{1}$$

Note that all optical radii (and the mirror radius) are positive in the example diagrammed alongside. The value of m depends on which side of the hologram is being illuminated; $m = +1$ if the illumination is coming from the same side as the reference beam—there is no other diffracted order if the hologram is reasonably thick. However, the physical mirrors in the hologram layer are indeed curved, and illuminating the hologram from the opposite side produces the effect of a concave mirror instead of convex, which is usually a real image focused in space.

The optics of in-line reflection holograms: angles

The reflecting fringes in the holograms we are considering here are parallel to the emulsion surface (ignoring their curvature for the moment), because the rays that create them are incident at equal but opposite angles (zero degrees, in this case). These are called "conformal fringes" because their shape conforms to that of the emulsion surface, which allows them to act like mirrors in their geometrical properties too. That is, that the diffracted light leaves the hologram as though it were reflected by a flat mirror parallel to the surface, independent of the wavelength that is reflected.



$$\theta_{out} = 180° - \theta_{ill} \tag{2}$$

Emulsion swelling effects:

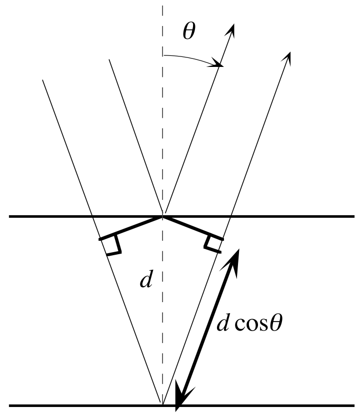
One of the interesting properties of reflection holograms is the effect of swelling or shrinkage of the emulsion during viewing. If you breath upon the processed hologram (so as to condense water upon the emulsion, which is quickly absorbed by the sponge-like gelatin layer to swell it slightly), the reflected color red-shifts to a longer wavelength. Conversely, heating the hologram (with a hair drying, for example) to dry it out and shrink it a bit will cause a blue-shift to a shorter wavelength. Depending on how the hologram was processed (especially on whether it was cross-linked or hardened), it will show these effects more or less strongly. For exposure and illumination perpendicular to the plate, the reflected wavelength will vary according to

$$\lambda_{2,0} = \frac{n_2 \cdot t_2}{n_1 \cdot t_1} \lambda_1 \cdot \text{(angle effects, to be discussed)} \tag{3}$$

where t_1 and t_2 are the physical thicknesses of the emulsion during exposure and viewing, and n_1 and n_2 are the refractive indices of the emulsion during exposure and viewing. All of these can be controlled to some extent—one popular method of creating colors different from that of the exposing laser is the pre-swelling of the emulsion by imbibition of solutions of sugar and water, which wash out during processing and produce a controllable blue-shift effect. These are called "pseudo-color" processes, because the colors are not actually those of the objects portrayed.

Viewing angle effects: the "blue shift"

Another effect that is easily observed is the variation of image color as the hologram is tipped away from the illuminating beam, so that the angle of illumination increases from zero to some value. The reflected image blue-shifts in color, although the color shift may be too small to notice in a deep-red image. The reason is that the time delay between the light reflected by adjacent hologram layers actually *decreases* when the light comes in at an angle, which is the opposite of what you might expect.



To examine this unusual behavior, consider the sketch just above. The key to understanding the effect is to note that it is the extra distance that the second reflection must travel that matters, but that the "race" between the two reflections begins at the point where they are last "abreast" and ends when they cross another line that is perpendicular to both rays. There are two effects: the second ray does spend more "time" between the reflecting layers as the illumination angle increases, but it also "cuts a corner" with increasing angle, and this effect dominates to produce the net decrease of delay between the reflections. The delay produces a wavelength shift given by

$$\lambda_{2,external} = 2 \cdot n_2 \cdot d \cdot \cos\theta_{\text{int}} \quad . \tag{4}$$

The result is that, contrary to some people's intuitive expectation, a reflection hologram image "blue shifts" as it is illuminated at steeper angles.

Conversely, if the hologram is exposed at an angle and viewed perpendicularly, the reflected wavelength is longer than the exposing wavelength: a "red shift" occurs. Assuming that the object and reference beams are coming in at opposite angles, so as to produce fringes that are parallel with, or conformal to, the hologram surface, and that the illuminated beam reflects at the mirror angle. That is:

$$\theta_{\text{ref,ext}} = 180° - \theta_{\text{obj,ext}}, \theta_{\text{ill,ext}} = 180° - \theta_{\text{out,ext}} \quad . \tag{5}$$

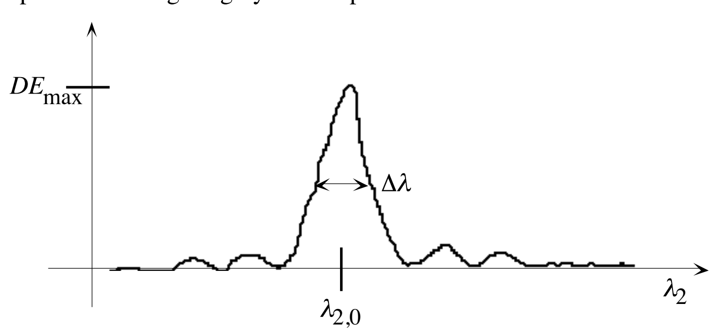
then the fuller version of Eq. 3 becomes:

$$\frac{\lambda_{2,\text{ext}}}{\lambda_{1,\text{ext}}} = \frac{n_2 \cdot t_2 \cdot \cos\theta_{\text{ill,int}}}{n_1 \cdot t_1 \cdot \cos\theta_{\text{ref,int}}} \quad . \tag{6}$$

Diffraction efficiency of reflection holograms:

[insert]discussion of thickness $Q = 2 \cdot \pi \cdot \lambda_2 \cdot T \,/\, n_2\ (\text{spacing})^2$]

The amount of light that an "in-line" reflection hologram will reflect depends on more factors than for transmission holograms, especially on the wavelength of the light involved. A typical reflection hologram will have a reflectance or diffraction efficiency spectrum having roughly this shape:



where $\lambda_{2,0}$ is the central wavelength of the reflection spectrum, which we have seen above is determined by the exposing wavelength, shrinkage effects, and the angle of incidence. The maximum height of the curve, or DE_{max}, and the width of the reflection spectrum at its half-height points, $\Delta\lambda_2$, are what we are about to discuss. We have to be content with a low-efficiency-only model here, although it can be extended to more practical levels with more advanced mathematics.

First we will consider the amount of reflection at the spectrum's peak. We will model the reflection hologram as a stack of alternating layers of high and low refractive index, n_{low} and n_{high}, each of equivalent thickness of one-quarter wavelength. Each high-to-low or low-to-high boundary will reflect a small amount of light amplitude, as given by the venerable equations of Fresnel (this simple form is for perpendicular incidence):

$$R_{ampl} = \frac{n_{high} - n_{low}}{n_{high} + n_{low}} \quad . \tag{7}$$

Note that a transition from high- to low index gives a positive reflection, while a transition from low to high gives a negative reflection. However, because there is a round-trip delay of one-half wavelength between them, they arrive in-phase and add together.

Now the wavelength inside the emulsion is λ_0/n_2, and if the emulsion is of thickness t_2, there will be a total of

$$M = \frac{t_2}{\left(\dfrac{\lambda_{2,0}}{2n_2}\right)} = \frac{2n_2t_2}{\lambda_{2,0}} \tag{8}$$

each of the high- and low-refractive-index layers. Assuming that the reflectance of each layer is low enough that we can ignore the effects of double reflections (the Born single-scattering approximation), the amount of light reflected by each transition will be equal and the total reflected amplitude will be:

$$R_{ampl,total} = 2MR_{ampl} = M\frac{n_{high} - n_{low}}{\left(\dfrac{n_{high} + n_{low}}{2} = n_2\right)} \tag{9}$$

It remains only to slightly adjust the value of the refractive index modulation to take into account that the actual variation is nearly sinusoidal instead of a step-like variation, and has a peak-to-peak modulation of Δn_2. The ratio is the same as that between an electrical square wave and its fundamental frequency:

$$\left(n_{high} - n_{low}\right)_{\text{square-wave}} = \frac{\pi}{4}\Delta n_{\text{sinusoid}} \tag{10}$$

Thus the total intensity reflectance, or diffraction efficiency, can be approximated by:

$$\begin{aligned} DE_{+1} &= \left|R_{ampl,total}\right|^2 = 0.62\ M^2\left(\frac{\Delta n}{n_2}\right)^2 \\ &= \left(\frac{\pi}{4}\frac{t_2 \cdot \Delta n}{\lambda_2}\right)^2 \end{aligned} \tag{11}$$

As Δn and thus DE_{+1} increase, this simple model fails to account for all the important phenomena, especially the dropoff of the illumination as light propagates deeper into the hologram, and the multiple reflections of the diffracted light. The next level of complexity was provided by the analysis of the "coupled wave model" by Herwig Kogelnik in 1969[3]. The result shows that as the modulation, Δn, further increases, the diffraction efficiency stops growing so quickly, and eventually rolls over to asymptotically approach its maximum value of unity. The analytical expression for the diffraction efficiency includes the hyperbolic tangent function, $\tanh(x)$:

$$DE_{+1} = \text{(to be continued...)} \tag{12}$$

Spectrum width:

For many applications of reflection holograms, it is important to keep the width of the reflected spectrum to a minimum. The simple low-diffraction-efficiency model gives a FWHM (full-width at half-maximum) bandwidth of roughly

$$\Delta\lambda = \frac{\lambda_{2,0}}{M} \tag{13}$$

One way to understand this number is to recall that, at the central maximum, the reflections from all the *M* layers arrive in-phase. In particular, the reflection from the back layer arrives $(M-1)\cdot 2\pi$ delayed with respect to the first layer. But at, for example, a shorter wavelength (say $\lambda_{2,L} = \lambda_{2,0} - \Delta\lambda$), there will be an added phase delay of

$$\begin{aligned} \Delta\phi &= 2\pi \frac{2n_2 t_2}{\lambda_{2,0} - \Delta\lambda} - 2\pi \frac{2n_2 t_2}{\lambda_{2,0}} \\ &\approx 2\pi \cdot 2n_2 t_2 \cdot \frac{\Delta\lambda}{\lambda_{2,0}^2} \end{aligned} \tag{14}$$

If the phase delay from the back of the hologram is 2π, then there will be a layer near the center of the hologram for which it will be π, and the reflection from the front layer and the middle layer will cancel out. For the next layer into the emulsion, there will be another just beyond the middle layer that will cancel it out, and thus there will be cancellation by pairs throughout the depth of the emulsion, and the total reflection (that is, the diffraction efficiency) will drop to zero.

Substituting the key variables back into the equations yields the expression for the required wavelength shift as:

$$\Delta\lambda = \frac{\lambda_{2,0}^2}{2n_2 t_2} = \frac{\lambda_{2,0}}{M} \tag{15}$$

Some analyses will produce an M+1 or M–1 in the denominator, but because *N* is typically more than ten, we will ignore this detail. We also assert that the distance from the spectral peak to the first zero will be the same as the FWHM spectral width, to within the accuracy needed for our purposes.

Note that increasing the thickness of a hologram both decreases the spectral blur in white-light illumination and increases the peak diffraction efficiency. This would suggest that making reflection holograms very thick indeed would be a good idea, and yet this is usually not the case, for reasons that we will explore only briefly.

Anomalous spectral shapes:

It often happens that the reflection spectra of real holograms are very different from the idealized “sinc-squared” spectrum that a simple Fourier analysis predicts. There are several reasons that we ought at least to be aware of:

1) the reflectance of the first few layers may be much higher than the simple theory can accommodate, or

2) the reflectance of the various layers may be very different due to the differing chemistries that occur as solutions diffuse through the emulsion during processing, or

3) similar depth-dependent chemistry changes may cause an uneven swelling or shrinkage of the hologram through its depth, which is often described as a “chirping” of the hologram’s fringe spacing or spatial frequency (one indicator of #2 and #3 is that the hologram may look very different when viewed from the emulsion side and the support side).

For some applications (hologram jewelry made with dichromated gelatin is one example), a deliberately wide spectrum with high diffraction efficiency is desired, in order to give a silvery or golden image effect. Very dramatic processing is used, involving baths of hot alcohol and vigorous washing, to give the combination of high refractive index change and exaggerated chirping that is needed.

Conclusions:

to be continued...

References:

1. Yu.N. Denisyuk, “Photographic reconstruction of the optial properties of an object in its own scattered radiation field,” *Soviet Physiks Doklady*, **7**, pp. 543-545 (1962).
2. Yu.N. Denisyuk, “On the reproduction of the optical properties of an object by the wave field of its own scattered radiation,” *Optics & Spectroscopy*, **18**, pp. 365-368 (1963).
3. H. Kogelnik, “Coupled wave theory for thick hologram gratings,” *Bell Systems Technical Journal*, **48**, pp. 2909-2947 (1969).