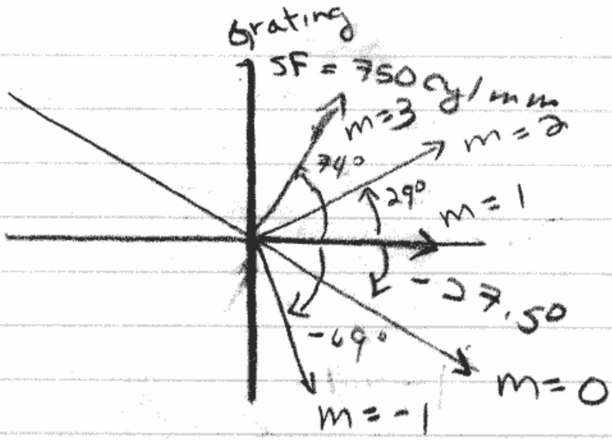
MAS450 Spring 2003 PS#2 solutions

1

a + b



$\sin\theta_{OUT} = m\lambda f + \sin\theta_{IN}$

$m\lambda f + (-.462)$

$\lambda = 633 \times 10^{-6}$ mm

$f = 750$ cy/mm

$\lambda f = .47$

Note: your values may be off from the ones here by Δ due to significant figures.

$m = 1 \rightarrow \sin\theta = .475 - .462 = .013 \rightarrow \theta = .745$ | $m = 1$

$m = +2 \rightarrow \sin\theta = 2(.475) - .462 = .488 \rightarrow \theta = 29.2°$ | $m = 2$

$m = +3 \rightarrow \sin\theta = 3(.475) - .462 = .963 \rightarrow \theta = 74.4°$ | $m = 3$

$m \geq +4 \rightarrow \sin\theta > 1 \rightarrow$ evanescent $\rightarrow m \geq 4$

$m = -1 \rightarrow \sin\theta = -.475 - .462 = -.937 \rightarrow \boxed{\theta = -69.6° \quad m = -1}$

$m \leq -2 \rightarrow \sin\theta < -1 \rightarrow \boxed{\text{evanescent}}$ $m \leq -2$

deflection:

$m = 1 \Rightarrow .745 - -27.5' = 26.8°$ $m = 1$

$m = +2 \Rightarrow 29.2 - -27.5°$ $56.7°$ $m = 2$

$m = +3 \Rightarrow 74.4° - -27.5 = 102°$ $m = 3$

$m = -1 \Rightarrow -69.2° - -27.5° = \boxed{-42.1°}$ $m = -1$

(1c) wave length change

$m=1$, $\lambda = 633\,nm \Rightarrow$ from before, $\boxed{\theta = .75^\circ\ (m=1, \lambda=633)}$

$m=1$, $\lambda = 540\,nm \rightarrow$

$\sin\theta_{out} = (540\times10^{-6}\,mm \cdot 750\,cy/mm) + -.462 = -.0570$

$\boxed{\theta_{OUT} = \underline{-3.27^\circ}\ (m=1, \lambda=540)}$

$\lambda = 470\,nm \rightarrow$

$\sin\theta_{OUT} = (470\times10^{-6}\,mm)(750\,cy/mm) + (-.462) = -.110$

$\boxed{\theta_{OUT} = -6.27^\circ\ (m=1, \lambda=470)}$

$\boxed{m=-1 \Rightarrow \lambda = 633\,nm \Rightarrow \theta = -69.2^\circ}$

$\lambda = 540\,nm \Rightarrow$

$\sin\theta_{OUT} = (-540\times10^{-6}\,mm \cdot 750\,cy/mm) + (-.462) = -.867$

$\boxed{\theta_{OUT} = \underline{-60.1}^\circ\ (m=-1, \lambda=540)}$

$\lambda = 470\,nm \Rightarrow$

$\sin\theta_{OUT} = (-470\times10^{-6}\,mm \cdot 750\,cy/mm) + (-.462) = -.814$

$\boxed{\theta_{OUT} = \underline{-54.5}^\circ\ (m=-1, \lambda=470)}$

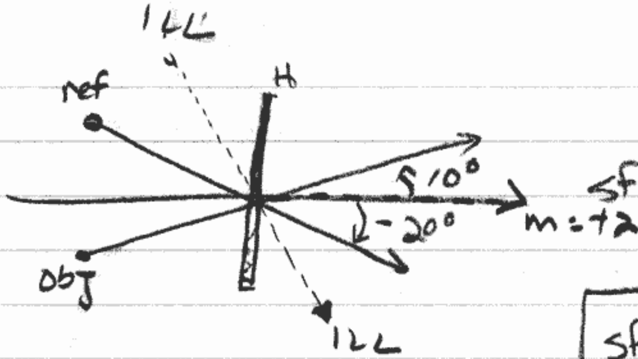
deflection angles:

$m=1$	$m=-1$
$\lambda = 633\,nm \Rightarrow 26.8^\circ$	$\lambda = 633\,nm \Rightarrow -41.7^\circ$
$\lambda = 540\,nm \Rightarrow 24.2^\circ$	$\lambda = 540\,nm \Rightarrow -32.6^\circ$
$\lambda = 470\,nm \Rightarrow 21.2^\circ$	$\lambda = 470\,nm \Rightarrow -27.0^\circ$

when the input angle is non-zero, the diffracted output orders are asymmetric around $m=0$ for both increasing $|m|$ and changing wavelength.

also, "red rotates radically" → longer wavelengths are diffracted more (greater deflection)

(2)



$sf = \sin\theta_1 - \sin\theta_2$

$\boxed{sf = 810\,cy/mm}$

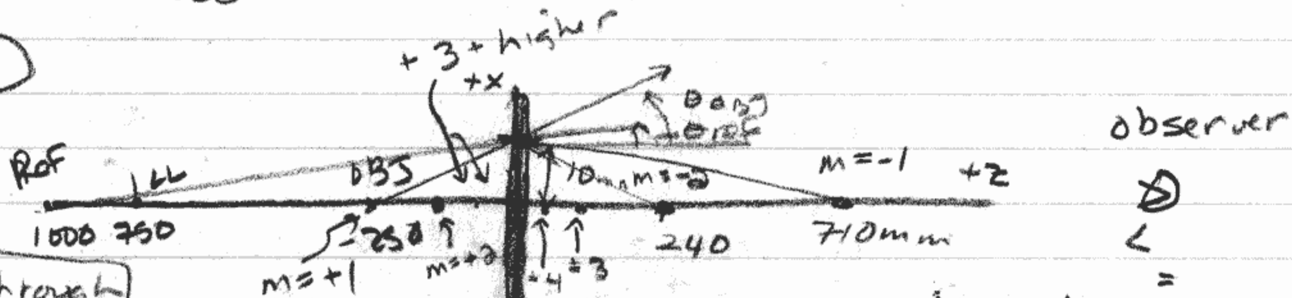
solve for $\sin\theta_{ILL}$

$$\sin\theta_{OUT} = m\frac{\lambda_2}{\lambda_1}(\sin\theta_{OBJ} - \sin\theta_{REF}) + \sin\theta_{ILL}$$

$$0 = (+2)\frac{543}{633}(\sin(10) - \sin(-20)) + \sin\theta_{ILL}$$

$$\theta_{ILL} = -62^\circ$$

(3)



ray trace through
x=10
and
x=0 ⇒ ZFP

$$\sin\theta_{OUT} = m\frac{\lambda_2}{\lambda_1}\left(=\frac{560}{633}\right)(\sin\theta_{OBJ} - \sin\theta_{REF}) + \sin\theta_{ILL}$$

$$m=1 \Rightarrow \quad m\,\frac{560}{633}\,(4.0\times10^{-2} - 1.0\times10^{-2}) + 1.333\times10^{-2}$$

$$\tan\theta_{OUT} \approx \sin\theta_{OUT} = m(2.654\times10^{-2}) + 1.333\times10^{-2}$$

$$\frac{10\,mm}{-z} \approx m(2.654\times10^{-2}) + 1.333\times10^{-2}$$

$$z \cong -\left[10\,mm \cdot \frac{1}{m(2.654\times10^{-2}) + 1.333\times10^{-2}}\right]$$

$m=1 \Rightarrow z \cong -250$ mm
$m=2 \Rightarrow z \cong -150$ mm
$m=-1 \Rightarrow z \cong +760$ mm
$m=-2 \Rightarrow z \cong +250$ mm

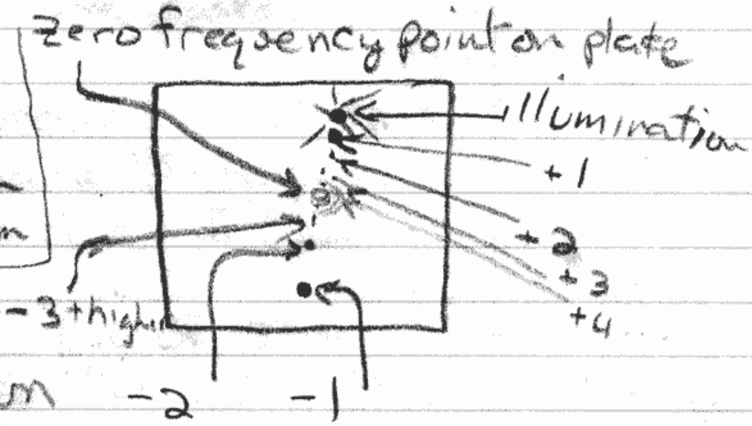


plate tipped
slightly down

(3c) use $\frac{1}{R}$ equation:

$R_{REF} = 1000\,mm$, $R_{OBJ} = 250\,mm$ $R_{ILL} = 750\,mm$

$\lambda_2 = 560\,nm$ $\lambda_1 = 633\,nm$

$$\frac{1}{R_{OUT}} = m\frac{\lambda_2}{\lambda_1}\left(\frac{1}{R_{OBJ}} - \frac{1}{R_{REF}}\right) + \frac{1}{R_{ILL}}$$

$$\frac{1}{R_{OUT}} = m\frac{560}{633}\left(\frac{1}{250} - \frac{1}{1000}\right) + \frac{1}{750}$$

$$\frac{1}{R_{OUT}} = m\left(2.654\times10^{-3}\right) + 1.333\times10^{-3}$$

$$R_{OUT} = \frac{1\,mm}{m(2.654\times10^{-3}) + 1.333\times10^{-3}}$$

you could work this eqn. for all $m = \pm 1, 2$, or you could observe that this equation is the same as you found before:

$$z \cong -\left[\frac{10\,mm}{m(2.654\times10^{-2}) + 1.333\times10^{-2}}\right]$$

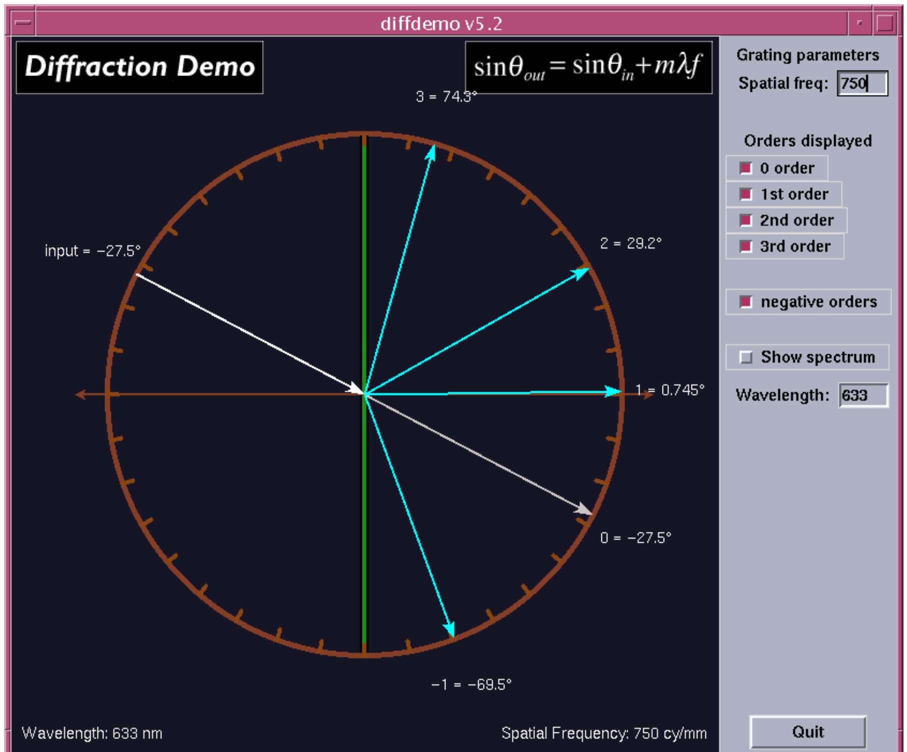
since the above result using raytracing used the small angle approximation, it follows that "$\frac{1}{R}$" has a similar dependence on small angles in some cases.

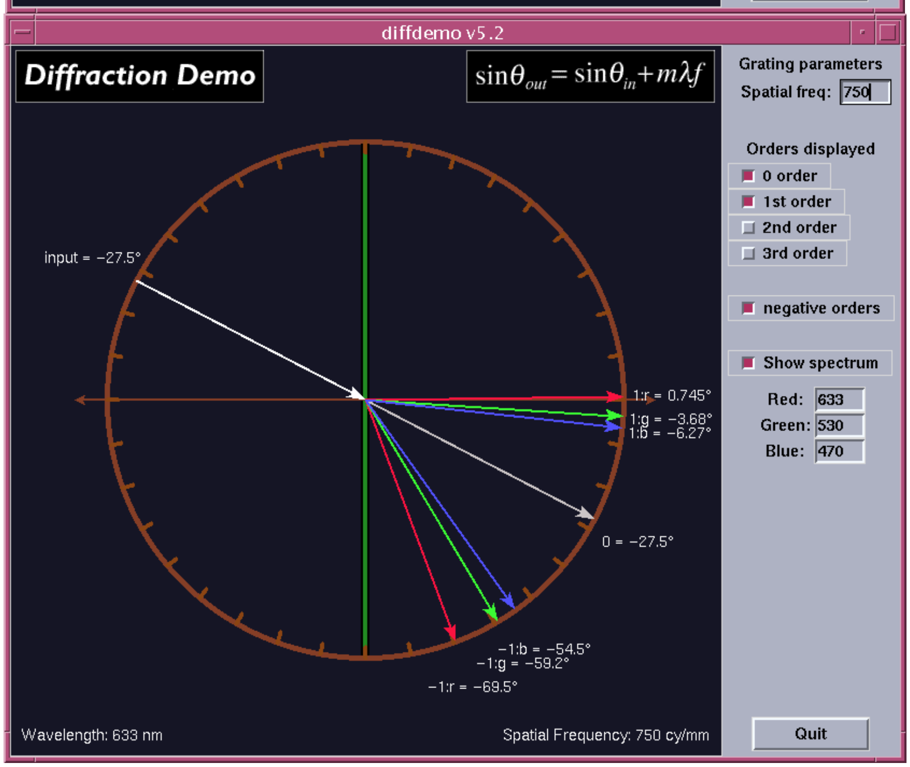
(3d) higher orders are located closer to the plate than their same-signed lower orders (as a general rule)

(3e) A viewer would see a line of sources with the brightest being the illumination (see prev. picture)

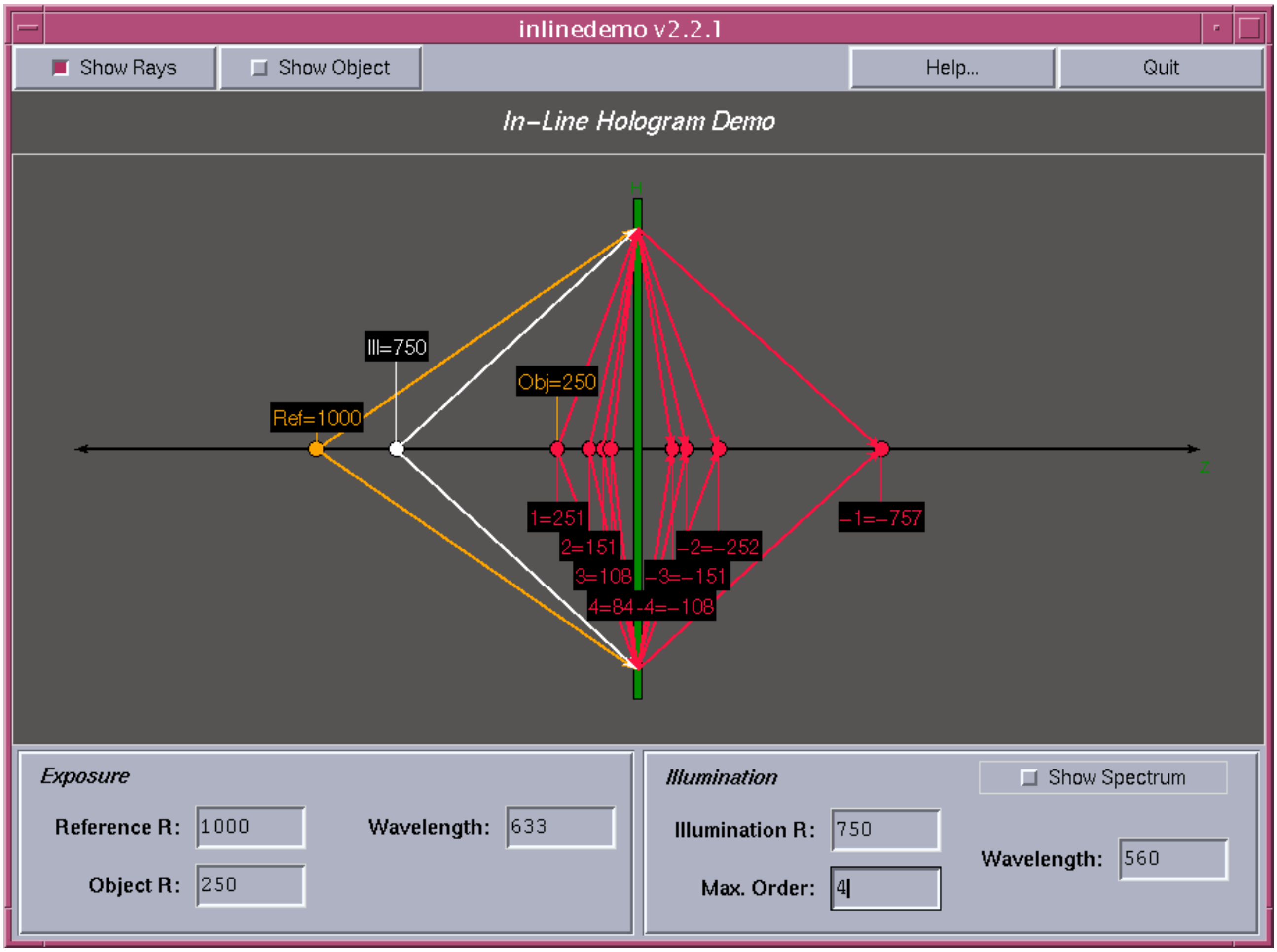
MAS.450 Problem Set #2 solutions. Your sketches should look something like these output images from the demo programs. NOTE: because of rounding and other slight incompatibilities, values may be slightly different than your values or those in the problem set solutions.

Problem 2:





Problem 3:


inlinedemo v2.2.1
Show Rays
Show Object
Help...
Quit
In-Line Hologram Demo
Ill=750
Obj=250
Ref=1000
1=251
2=151
3=108
4=84
-1=-757
-2=-252
-3=-151
-4=-108
Exposure
Reference R:
1000
Wavelength:
633
Object R:
250
Illumination
Show Spectrum
Illumination R:
750
Wavelength:
560
Max. Order:
4