Chapter 2: Light as Waves

What we think of as “light” is actually a ripple-like disturbance of combined electrical and magnetic fields (in the so-called “classical” or “non-quantum-mechanical approximation). As such, every good optics book dutifully begins with a discussion of Maxwell's equations, which can also be widely found on T-shirts around MIT[1,2,3]. The electric (and magnetic) fields are vectors, **E** and **H** respectively, indicating the directions between lower and higher electrical and magnetic potentials. Everything follows from these mathematical elaborations by James Clerk Maxwell of observations by Michael Faraday, that there is a coupling of the spatial variations of one of the fields (denoted by the “curl” or “div” of its vector) and the time variation of the other field, and vice versa (the first two equations). Either field may be stimulated—by a temporal variation of charge density in one case, and of current in the other—giving rise to a wave that immediately couples one to the other. Together, the electric and magnetic fields propagate away from the source like ripples in a pond with characteristic shapes that depend on how the disturbance was started, a manifestation of the *wave equation* that can be derived from Maxwell’s equations. In this chapter, we will first consider some aspects of the shapes of the waves, then their time variations, and finally some underlying aspects of the electromagnetic waves themselves.

Maxwell, James Clerk, 1831-1879, Scottish physicist who made fundamental contributions to electromagnetic theory and the kinetic theory of gases. See esp. *A Treatise on Electricity and Magnetism* (1873). Maxwell was elected a member of the Royal Society at age 14!

$$\nabla \cdot \mathbf{D} = \rho$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$$

$$\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{dt} = \mathbf{J}$$

Wave Shapes

The term “wave” really refers to “a self-propagating disturbance” such that a disturbance at some location, such as from a pebble dropped into a pond, produces a disturbance somewhere else at a later time, without any molecules of water actually traveling from the first place to the second. Physicists often refer to those familiar ripples in a pond when talking about waves, and use ripple tanks to illustrate their thoughts, but water-wave propagation is actually quite a complex problem, even in two dimensions. We will be concerned instead with light waves propagating in three-dimensional space, such as from the point-like focus of a laser beam. There are three simple shapes of light waves that will cover most of the cases we will have to deal with.

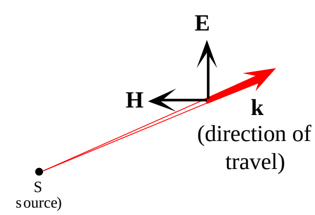
$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) E_x = \varepsilon_0 \mu_0 \frac{\partial^2}{\partial t^2} E_x$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) E_y = \varepsilon_0 \mu_0 \frac{\partial^2}{\partial t^2} E_y$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) E_z = \varepsilon_0 \mu_0 \frac{\partial^2}{\partial t^2} E_z$$

<u>Spherical Waves:</u>

If the wave source is a spark-like disturbance at an idealized point in space, say at $(x,y,z) = (0,0,0)$, then the resulting pulsed electrical and magnetic disturbances will spread out like a sphere, with the radius of the sphere increasing linearly with time at a rate we call the “speed of light,” which is determined by the properties of the medium (typically air, which is close enough to a vacuum for most purposes). The speed of light in a medium is given by one over the square root of the product of the dielectric constant and magnetic permittivity of the medium, and is equal to 3 x 10^8 meters/sec in a vacuum. In denser media, such as air, water, or glass, the dielectric constant increases and the waves slow down. The ratio of the speed in a vacuum to the speed in a particular medium is called the *refractive index* of that medium, about which we will hear more later. For the moment, let’s consider waves in a uniform medium with a refractive index of unity (space, or air).



$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \equiv 299,792,458 \text{ m/s } \left(3 \times 10^8\right)$$

$$= 186,282 \text{ miles/second}$$

$$speed = \frac{c}{n}$$

$$n = 1.33 \text{ for water}$$

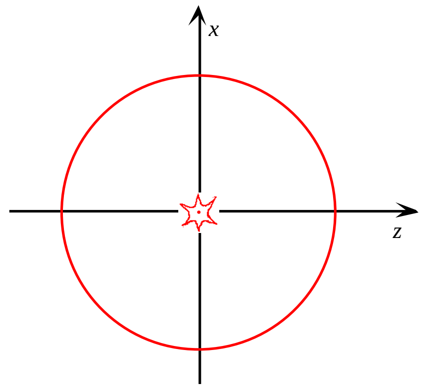
$$= 1.50 \text{ for glass, plastic}$$

$$= 1.000\,294 \ f(T^\circ) \text{ for air}$$

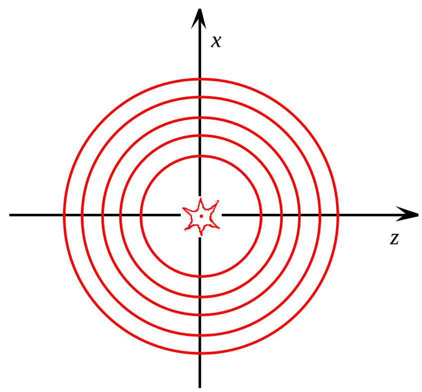
We have to be a little careful about the definition of the term “wavefront!” For a spark source, we can think of the wave as defined by a small interval in time when the electric field is non-zero, a “spike” in other words, at a single point in space. Maxwell’s equations say that the resulting pulse-like disturbance will move outward from that point at the same speed in all directions, forming an expanding sphere. The pulse intensity cannot actually be the same in all directions, but let’s imagine for the moment that it is (we are usually interested in only a fairly small range of angles anyway, where it can be virtually constant). As the wave spreads out, its amplitude drops as one over the distance (this will provide “conservation of energy,” to be discussed later on), but the “spike” stays a “spike” as it moves outward. We will think of the locations in space where we could observe the spike at time t as describing a sphere of radius r, where

$$r = \sqrt{x^2 + y^2 + z^2} = c \cdot t \ , \qquad (1)$$

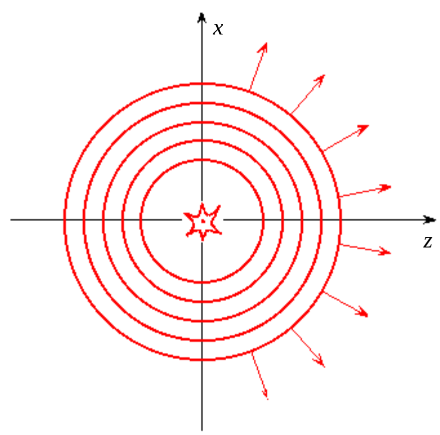
and c is the speed of light, mentioned above, and t is the time since the spark. This sphere is what we will call the *wavefront*. The disturbance moves quickly outward, always moving perpendicularly to the wavefront at every location, so that the radius of curvature of the spherical wavefront increases as the wave moves outward, and is the same everywhere on the wavefront.



What about "rays?" We sometimes think of a point of light as emitting imaginary particles outward, which travel at a constant speed, their trajectories described by straight lines called "rays." Our emphasis in this course will be on the description of light as a <u>wave</u> phenomenon instead, in which the relative time properties of the light energy headed in various directions becomes very important, which information the <u>ray</u> description generally loses. Nevertheless, we can draw arrows perpendicular to the spherical wavefront at any location and get a good prediction of where the energy will be found an instant later (except in *birefringent* crystals); it is these arrows that we will refer to as "rays" when we sketch what is going on in an experiment.



Sketches are important to optickers and holographers alike, and become problematical because we have only a two-dimensional paper surface to sketch them upon! Usually, these sketches will represent *slices* through a 3-D volume, although we will also attempt isometric-like views of a scene (usually a layout of optical components), which is a *projection* through a 3-D volume—quite a different kettle of fish (the differences will usually be clear from the context). In most of what we do, light will be traveling from left to right (considered to be good optical design practice), and we will adopt the z-axis as the horizontal axis, with the x-axis pointing upwards. So, we can attempt a sketch of a spherical wave as seen at a single instant of time as a "snapshot" of a slice-view of the spike, which simply looks like a circle.
If we take a sequence of such "snapshots" at equally-spaced intervals of time, we get a series of concentric rings, also equally spaced in radius.
The direction of energy propagation is everywhere perpendicular to the surface of the spherical wave, so the wavefront reproduces itself an instant later as a sphere of larger radius.



Mathematically, if we describe the source "pulse" as some function, $p(t)$, at the center, then the pulse arrives at a radius r after a delay time given by r/c, and falls off in strength as $1/r$. This can be expressed as an electric field of strength $E(r)$ given by

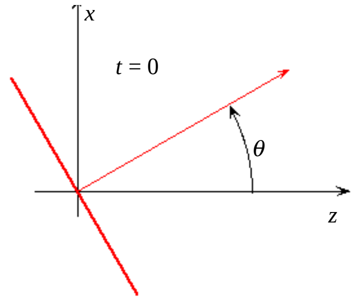
$$E(r,t) = \frac{1}{r}\, p\!\left(t - \frac{r}{c}\right) , \qquad (2)$$

or, in terms of the x,y,z coordinates

$$E(x,y,z,t) = \frac{1}{\sqrt{x^2+y^2+z^2}}\, p\!\left(t - \frac{\sqrt{x^2+y^2+z^2}}{c}\right) . \qquad (3)$$

<u>Plane Waves:</u>
After a spherical wavefront has propagated for a very long distance, its wavefronts become effectively flat or "planar" over the area of interest to us (which is to say that its radius of curvature has become nearly infinite), so we refer to them as "plane" waves. For example, the light from a nearby star (other than our own sun, which is too wide to be a point-like source) arrives as a plane wave (of course, if we changed locations in the galaxy, we would find that the angle of the plane wave would change, and that it is truly spherical). Thus plane waves are really an abstraction, but physicists are very fond of them for simplifying analyses, and we have to take them into account as an interesting limiting case of a spherical wave. Because the source location, which would ordinarily define the center of our optical coordinate system, is a long ways away, we refer instead to the local inclination of the wavefront as observed at the new center of our experimental coordinate system. We describe the plane

wavefront as having an angle, θ, measured between the wavefront normal and the horizontal or z-axis.

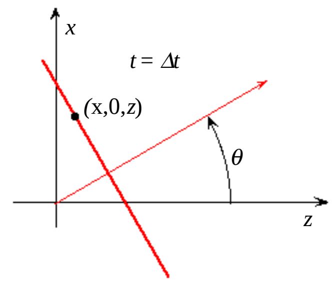


The location of this wavefront at time t=0, shown first, is given by the x,z locations of equal electrical voltage,

$$x = -z/\tan\theta \text{, or}$$
$$x\sin\theta + z\cos\theta = 0 . \tag{4}$$

A short time, Δt, later, the wavefront will have moved a distance $d = c\cdot\Delta t$ perpendicularly to itself, and the equation for the wavefront becomes

$$x\sin\theta + z\cos\theta = c\cdot\Delta t \; . \tag{5}$$



Mathematically, then, if the source pulse has the form $p(t)$ at the $(x,y,z) = (0,0,0)$ point, then the field seen at any other location (x,y,z) is retarded by d/c, where d is the shortest distance between the origin and the wavefront passing through the observation point (the perpendicular distance from the origin to the wavefront)

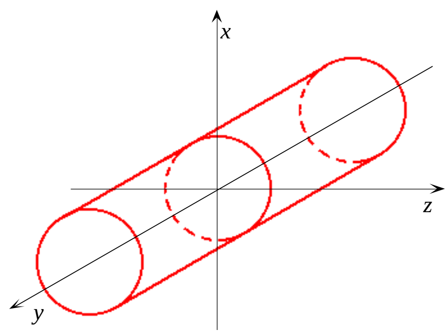
$$d = x\sin\theta + z\cos\theta \; . \tag{6}$$

The magnitude of the pulse does not diminish because the wavefront is no longer "spreading out" as it would for a spherical wave; the wave amplitude is constant as it moves ahead:

$$E(x,y,z,t) = p\left(t - \frac{x\sin\theta + z\cos\theta}{c}\right) . \tag{7}$$

Cylindrical Waves:

Later on, we will encounter waves that have different curvatures in different directions, called *astigmatic* waves. A simple first case of one is a cylindrical wave, which we can think of as propagating from a surge of current in an infinitely long wire; let's say the wire is stretched in the y-direction. The wavefront will lift off of the wire as a cylindrical tube, and propagate outward as a tube of constantly increasing radius equal to the speed of light times the propagation time. At some distance from the wire, let's say one meter, the wavefront will be curved around the wire in one direction, but not curved in the other. These are hard to sketch clearly, but an isometric-style projected view would look like this:

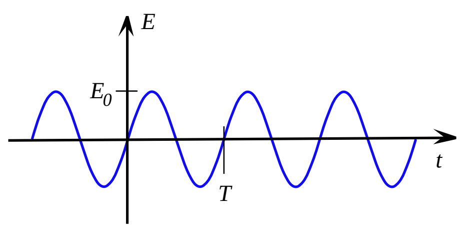


Mathematically, it would have a representation like

$$E(x,y,z,t) = \frac{1}{\left(x^2+z^2\right)^{\frac{1}{4}}}\, p\left(t - \frac{\sqrt{x^2+z^2}}{c}\right) . \tag{8}$$

Light as REPETITIVE Waves:

So far, we have considered a single pulse-like wave propagating through 3-D space, but visible light is a repetitive wave, which is what makes holography possible too! In the case of light, the pulses are smoothed out so that the electric and magnetic fields are smoothly varying functions of time. If we stood at a particular point in space and measured the electric field of a wave passing by (spherical or plane), we might observe a voltage as seen here:

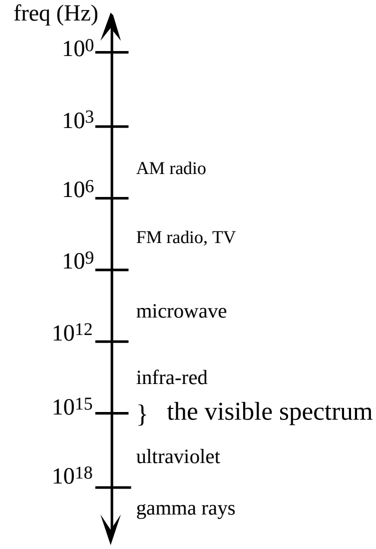


Mathematically, this is described by the trigonometric sine function, with time as its argument. Every T seconds (we call T the "period" of the wave), the argument increases by 2π or "full circle" (360°) and the voltage pattern repeats:

$$E(t) = E_0 \sin\left(2\pi \frac{t}{T}\right) . \tag{9}$$

The sine function derives its name from the sinuous "look" of the curve, which describes the x-coordinate of a point on the rim of a wheel as it turns through 360° or 2π radians.

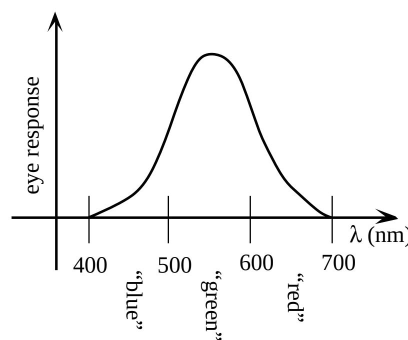
You might ask "Why are the waves sinusoidal, instead of saw-toothed or triangular?" The answer is, approximately, that the waves are generated by electrons oscillating at the ends of "springs" that represent the change of energy as the electron's orbit moves nearer to and farther from the nucleus. The actual process quickly gets into quantum complexities that we don't have time to deal with here! Similarly, our eyes respond only to the sinusoidal components, because the sensing structures are resonantly tuned. This all turns out to be handy, because the techniques of mathematical physics have largely been developed around sinusoidal signals since the days of Fourier, the extraordinary French mathematician and physicist.



As we move the observation point further from the source, the receipt of the wave is delayed a little by the extra propagation time, which causes an apparent shift in the sinusoidal wave by some angle, which we call the *phase* of the wave, and about which we will say much more later on. The strength of the wave also drops off a little, as the $1/r$-law dictates.

The rate of repetition is the only thing that separates light waves from radio, television, microwave, and gamma waves! The mathematical physics of all these varieties of electromagnetic waves are the same, but their practical and physiological properties are quite different. We think of the various waves as being arrayed in terms of their "frequencies," measured in cycles per second (called Hertz, Hz). Their "period" is what we have already seen as T, measured in seconds (or microseconds, or attoseconds). Their frequency is given by ν (the Greek letter "new", in cycles per second), where $\nu = 1/T$. Electrical engineers like to speak in terms of the "radian frequency," $\omega = 2\pi\nu = 2\pi/T$ ("omega," in radians/second), but we will speak strictly of the "natural frequency," ν, in these discussions.

The electromagnetic spectrum is described in most physics books, and we will outline it only briefly here. Suffice to say that the principles of holography apply to all frequencies of waves, not just visible light.



Of the entire electromagnetic spectrum, only a tiny sliver, less than a two-to-one range of frequencies (compared to the nine octaves of the audio spectrum), serves to evoke a response in the human eye that we call "seeing." Within this visible part of the spectrum, different regions evoke quite different sensations, which we distinguish by the term "colors." For unknown reasons, optickers like to describe the visible spectrum in terms of the wavelengths in a vacuum of the radiations that are involved. These wavelengths vary between 400 and 700 nanometers (nm, 10^{-9}m), and it is the extreme shortness of these wavelengths that accounts for many of the practical problems of making holograms. Listing the radiations of a few common lasers by frequency and wavelength gives us a chance to compare them.

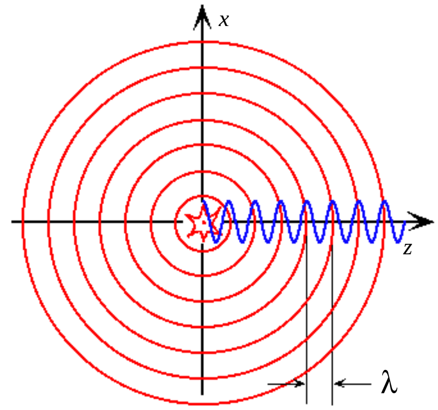
The sensation of color produced by light of various wavelengths (when viewed as an isolated spot in a dark surround) varies in a fairly reliable way as the wavelength varies from long to short. The color names of "red, orange, yellow, green, blue, and violet" and so forth are associated with various regions of the spectrum for that reason. We will simplify matters by referring only to the "red", "green," and "blue" areas, which will serve as additive color primaries.

Fourier, Jean Baptiste Joseph, Baron, 1768-1830, French mathematician and physicist, noted for his researches on heat diffusion and numerical equations. He originated the Fourier series, which allowed discontinuous functions to be represented by an infinite series of sines and cosines. See esp. *The Analytical Theory of Heat* (1822). Also, Governor of Lower Egypt for Napoleon.

Light as <u>Sinusoidal</u> Waves:

Now, mingling the wave <u>shapes</u> from our discussion of pulsed waves, and the sinusoidal <u>repetitiveness</u> of ordinary light, we can come up with a combined description of light in a form that can readily be manipulated in mathematical terms. Again we refer to illustrative sketches as capturing a "snapshot" of the wave, but this time the concentric circles represent the successively-emitted peaks of the repetitive sinusoidal waves (not a succession of snapshots, as before). The separation of the circles is the distance that the wave travels in T seconds, one cycle of the vibration, and is called the “wavelength,” designated by λ (the Greek letter “lambda”), so that $\lambda = \mathrm{c} \cdot T = \mathrm{c}/\nu$, Then we can write, for a <u>spherical</u> wave:

$$E(r,t) = \frac{E_0}{r}\sin\left(2\pi\frac{\left(t-\frac{r}{c}\right)}{T}\right) = \frac{E_0}{r}\sin\left(2\pi\left(\nu t - \frac{r}{\lambda}\right)\right) \qquad (10)$$
$$= \frac{E_0}{r}\sin\left(2\pi\nu t - \frac{2\pi}{\lambda}r\right)$$

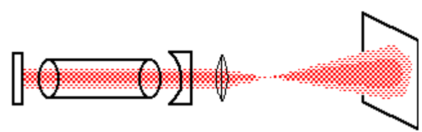


When we go on to consider plane waves, the situation is not much different: just plug in the new form of the pulsing function into the same old wave shape formula, and the new function results:

$$E(x,y,z,t) = E_0\sin\left(2\pi\frac{t - \frac{x\sin\theta + z\cos\theta}{c}}{T}\right)$$
$$= E_0\sin\left(2\pi\left(\nu t - \frac{x\sin\theta + z\cos\theta}{\lambda}\right)\right) \qquad (11)$$

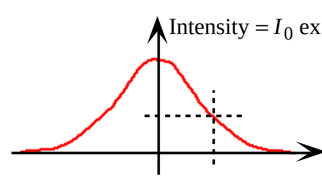
Coherence in waves

Our simple model of laser light assumes that it emerges from an ideal point source (the focus of a microscope lens, for example, as shown in the sketch). Within the diverging beam are the concentric spherical wavefronts, invisible to the eye. We also assume that this light has a perfectly well-defined and unvaryingly constant frequency. But both of these assumptions simplify the behavior of real lasers in ways that we should at least acknowledge—before continuing to ignore them for the most part! The term used to describe these properties of light is their *coherence,* and it has two “dimensions,” the *spatial* coherence, which describes the departure from ideal point-source-like behavior, and the *temporal* coherence, which describes the departure from ideal single-stable-frequency behavior. Both of these follow from the physics of resonant laser cavities and light amplifying media, which allow several oscillating modes along the direction of the resonator and from side to side.



<u>spatial coherence - point sources?</u>

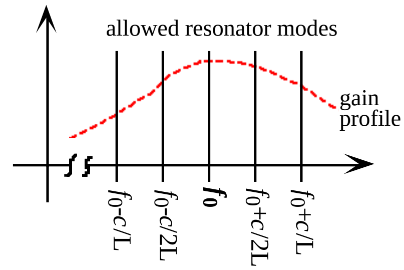
Laser cavities can, if nothing is done about it, resonate in a wide variety of modes, each with a slightly different frequency and spatial distribution [4]. Perhaps you know that mechanical structures also vibrate in different spatial modes (e.g., the sound of a drumhead depends on where you strike it)—these are easily seen with holographic interferometry! We usually distinguish between the various lateral or side-to-side modes, and the various longitudinal or along-the-cavity modes. The lowest-order or preferred longitudinal mode is the so-called TEM_{00} mode (“t-e-m-zero-zero”), which produces a nice bell-shaped output beam, called a *Gaussian* beam after the German mathematician who first used the exponential function involved (rhymes with “house-ian”).

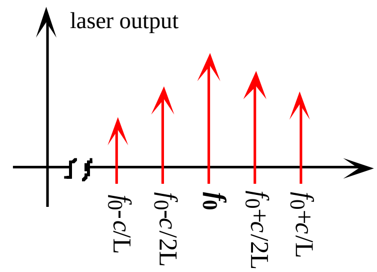


Gauss, Carl Friedrich (gaous), 1777-1855, German mathematician, physicist, and astronomer; b. Johann Friederich Carl Gauss. Considered the greatest mathematician of his time and the equal of Archimedes and Isaac Newton, Gauss made many discoveries before age twenty.

Other low-order lateral modes produce donut-shaped beams, two-lobed beams, and four-leaf-clover-shaped beams. If they are present, then the spot focused by a microscope objective will be larger than expected from a single zero-order mode. However, almost all lasers used today are “single mode” type, producing only the Gaussian beam profile. But if the laser system becomes overheated, or mechanically distorts for any reason, it can easily produce other low-order modes that will degrade its operation for holographic purposes. The main problem caused by the other modes is that their frequencies are significantly different than the lowest-order mode, which decreases the coherence length of the laser light, discussed below.

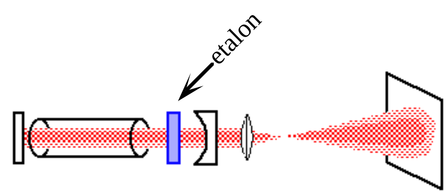
temporal coherence - monochromaticity?

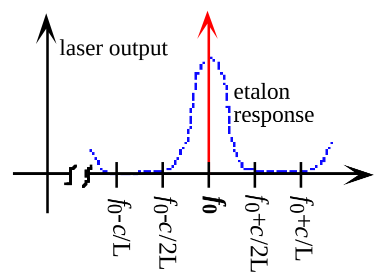




Usually, “monochromatic” means that a beam of light produces a single color (“red” for a helium-neon laser), as seen by the human eye. When talking about lasers, though, monochromatic usually means single frequency. Even when a laser is operating “multi-mode,” all of the output beams are the same color! You may know that resonators such as organ pipes and violin strings can be overblown or excited so as to produce overtones or higher harmonics, usually integer multiples of the lowest allowed or fundamental frequency (for which the string is a half wavelength long, for example). Typical laser resonators are a million half-wavelengths long, and are operating at extremely high harmonics of the basic frequency, given by $f_{cavity} = c/2L$ (in the range of a hundred megahertz). The laser’s amplification medium is capable of providing gain over a fairly wide range of frequencies, depending on exactly what the material and conditions are. Thus the resonator can be simultaneously operating at several nearby harmonics of the basic cavity frequency. The combination of these modes appears like a single output signal that is fluctuating in amplitude and frequency very rapidly, returning to the same frequency every round-trip cavity time (one over the fundamental cavity frequency).

Because the output frequency is fluctuating so widely, only light that emerges from the laser at nearly the same instant can interfere with itself—light that comes out at a little later time will produce an unsteady interference pattern that will average to zero contrast over a very short exposure time. The acceptable delay between light samples is usually expressed as the *coherence length* of the laser, the distance that light travels before the frequency changes so drastically as to destroy the interference pattern. For typical helium-neon lasers, the coherence length is somewhere between 100 and 150 mm (four to six inches). A holographic image of a scene will gradually lose brightness for components deeper than 50 to 75 mm from the object point whose path length has been carefully matched to that of the overlapping reference beam.





The coherence length can be dramatically increased by the use of an *etalon* in the laser cavity. This is typically a piece of very carefully polished glass with partially-reflecting coatings on each surface. Because it is only 10 mm or so thick, its cavity frequency is quite high, and its harmonics are deliberately separated by more than the width of the laser medium’s gain bandwidth. If an etalon harmonic can be aligned with the central resonance of the cavity, only one output frequency will be allowed, and it can have roughly 50% of the power previously put out in the collection of frequencies. This produces true single-frequency operation, and the coherence length can become many hundreds of meters. However, the system is still vulnerable to mechanical vibrations, which alter the separation of the main cavity mirrors, and thermal drift (which does the same thing). Thus, although manufacturers cite some amazing coherence lengths, they have to be measured over the time of a holographic exposure to be useful predictors, and can be much shorter in practice. These days, almost all medium- and large-frame lasers for holography include etalons (single-frequency operation makes life sooo much easier!), but we will still have to worry about it with helium-neon lasers.

laser speckle

Another quality of laser light that you have no doubt noticed by now is the gritty or sandy appearance of the surfaces that it illuminates. We call this grit “laser speckle.” It is an interference phenomenon that arises from the microscopic randomness of surfaces that look to our eyes like flat and smooth surfaces, and as such we can’t say much about it before we start looking at interference in more detail. Even then, the statistical techniques required go beyond the scope of this introductory course [5]. But we can at least start cataloging some interesting properties of laser speckle, so that we can know what to look for:

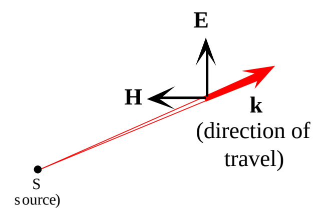
1. Laser speckle is always in focus; that is, its contrast is high no matter where our eyes are focused.

2. Laser speckle follows our motion. Or rather, it seems to stand still at whatever plane our eyes are focused at. Speckle can be a useful way of checking the accuracy of your eyeglass prescription!

3. The size of the speckles increases if the diameter of the pupil decreases—such as by looking through a pinhole.

We will have plenty of experience with laser speckle before the semester is over. Interestingly, it will gradually disappear for most of us!

E&M nature of the waves



Now we have to deal with some of the realities of the electromagnetic nature of these waves. Firstly, the electric field is a vector quantity, so we should designate it as a bold-face variable, **E***(x,y,z,t)*, and the vector's direction is always perpendicular (or "transverse") to the direction of propagation (except in some crystals). The magnetic vector is also transverse, and also perpendicular to the electric vector. Often, the electric vector vibrates up and down, or at some angle, so that its end point traces out a straight line. Such light is called "linearly polarized." In other cases, the tip of the electric vector may sweep out a circle or ellipse, and the light is called "circularly polarized" or "elliptically polarized." Light from incandescent bodies, such as the sun or electric lamps, varies its polarization state every few femtoseconds, and is called "unpolarized." But laser light is usually very well polarized, and is usually linearly polarized. The direction depends on the orientation of the Brewster windows for a gas laser, and is customarily vertical. Maxwell's equations require light to be a transverse vibration, which means that no point source can radiate equally in all directions; there have to be some directions of no radiation (for the same reason that you can't comb a hairy basketball flat—there must always be some cowlicks).

Polarization will come to be fairly important—two reasons come to mind:
1) The strength of the reflection of light from a glass surface depends on the polarization of the light (unless the light is coming in perpendicular to the surface), and
2) Only the parallel polarization components of two waves can combine to produce interference patterns (which we discuss in Ch. 3). Perpendicularly polarized beams (or rather, *orthogonally* polarized beams, in the general elliptical polarization case) cannot interfere at all.

Intensity (irradiance)

When it comes time for a light beam to do some work, such as expose a piece of photo film, we have to consider where the necessary energy comes from. In virtually all cases, it is the electric field that does the work; the magnetic field is just "along for the ride." Electrical gadgeteers know that the power absorbed by a resistive load is proportional to the average of the square of the electrical voltage across the load, divided by the resistance of the load. Similarly for optical power, which we usually measure in watts per square centimeter and call the "irradiance" or the "intensity" of the light (the latter being an obsolete term in the metric system, but still very commonly used)—it is proportional to the time average of the square of the electric field amplitude. Radiometry and photometry are baffling topics, as are most forms of accounting; they account for "what happened to the photons, or the lumens, that came out of the laser?" Suffice to say that if a uniform light beam has an electric field of the form

$$E(x,y,t) = A\ \sin(2\pi\nu t)\ \text{volts/meter}\ , \qquad (12)$$

then its average squared value will be

$$\left\langle |E(x,y,t)|^2 \right\rangle_{\substack{\text{time}\\ \text{average}}} = \frac{A^2}{2}\ , \qquad (13)$$

and it will deliver an irradiance of

$$I(x,y) = (\varepsilon_0 c)\left\langle |E(x,y,t)|^2 \right\rangle_{\substack{\text{time}\\ \text{average}}} = \left(\frac{\varepsilon_0 c}{2}\right) A^2 \quad , \qquad (14)$$
$$= 2.65 \times 10^{-3} E_{\text{rms}}^2 \text{ watts/meter}^2$$

in the MKS system of metric units. Full sunlight provides about one kilowatt per square meter, from which you can estimate its peak electric field!

Non-linear detection

All detectors (photocells, photo film, photodiodes, etc.) produce a current (electrons per second) that is proportional to the power in a light beam (which is proportional to the number of photons per second). The sensitivity may vary over the electromagnetic spectrum, but the linear electrical output is always proportional to the square of the optical input (the light amplitude). Most optical engineers have thought of irradiance as the linear input variable, but for coherent-light optickers, the amplitude of the wave is the important linear variable! It is the "square-law detection" (that is, non-linear detection) of this signal that causes many of the effects that seem so strange about coherent optics!

Intensity, Power, and Energy

Holographers often meter their beams, and it is important to understand what the various units of measurement are, and what they mean. Also, it is prudent to start thinking about the **safe** use of lasers, and this also requires understanding the various measures of laser light, and how they might effect a recording film or your eyes. There is nothing dangerous about the way we will be using lasers in this course's holography laboratory, but they are certainly capable of being misused with unhappy results.

		TIME	
		per second	
SPACE	per sq. cm	intensity (mW/cm^2)	exposure (ergs/cm^2)
	total area	power (mW)	energy (Joules)

There are different terms to describe whether we are measuring a light beam over a small area within the beam, and are interested in its energy density, or over the beam's entire area, and are interested in its total "flow." Similarly, we have to distinguish between a measurement of a rate of flow at a particular instant, or the cumulative flow over the entire length of a pulse or of an exposure time. The chart to the side notes the various terms. We will walk through them one by one.

1 μW-sec = 10 ergs

The power of continuous-wave lasers, such as the He-Ne lasers in the lab, is typically rated in milliwatts (perhaps between 1 and 10). However, if the beam is spread out with a lens, the "heat" felt by our hands will be proportional to the "intensity" or "irradiance," the power per unit area. And if this is totaled up over time, we will determine the total amount of "cooking" each small area of our hands have suffered, their "exposure" in milliwatt-seconds per unit area, also called milliJoules per square centimeter (photo film sensitivity is typically measured in ergs/cm^2, a cgs unit; an erg is 1/10,000th of a milliwatt-second). We add that our lasers are too weak to feel with your hand (go ahead, try it!), and that a typical flashlight emits about 60 mW of light (which is, of course, more spread out that a laser beam).

If you are dealing with a pulsed laser, such as a ruby laser, you will instead be told its total energy output per pulse, in Joules (watt-seconds). A one Joule laser is pretty big, and puts out as much light in a few tens of nanoseconds as a 10 mW He-Ne does in a minute and a half. If you divide the Joules by the number of square centimeters of the spread-out beam (and multiply by 10,000,000 to go from Joules to ergs), you will get the exposure of a piece of film put there. The danger of pulsed lasers comes from the very high instantaneous power of the beam at its peak, which may cause explosive damage to surfaces. A one-Joule laser with a 30 nanosecond-wide pulse reaches a peak power of 33 billion milliwatts. You would feel, hear, and remember that one! We won't use pulsed lasers in this course.

We will come back to these ideas when we make measurements for holographic exposures, which will involve overlapping beams, but the concepts will be the same. This discussion has been in terms of the "thermal" or radiometric power of a laser beam; a whole other set of units and measurements is used to describe its “brightness” or photometric power (lumens replace photons, for example). Photometry will get short shrift here, but we will have to consider it briefly later on in the course.

Conclusion:

We have skimmed through a lot of optics to find the mathematical descriptions of spherical and plane sinusoidal waves, which will serve us in good stead in the chapters just to come. You should make sure that you follow the logic that leads to the terms in the parentheses so far, as they will soon mutate into still further and more complex forms! Once those are under control, we may not often worry about the formalities of describing waves in detail, unless we are interested in the details of holographic optical element design. Likewise, there are lots of details about measuring the intensities of optical beams that we should know about, but only a few calculations that we will make over and over again. Nevertheless, as holography takes on new and different forms, there are likely to be times when we have to worry about measuring beams based on fundamental principles. For example, we have ignored the effect of exposure angle on the necessary dose for a holographic material—this is acceptably accurate for typical angles, but requires re-examination when we start talking about edge-lit holograms that involve very large beam angles.

References:

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