Chapter 14: White-Light Transmission “Rainbow” Holograms

A revolution in holography:

During the 1970s, two things happened that caused a revolution in display holography: the development of white-light viewable transmission holograms, and the development of very inexpensive processes to manufacture and distribute them. Both of these had their roots in the late sixties, and reached their full flower in the eighties, but the seventies were a time when everyone realized that important pieces of “the holography puzzle” were coming together to make display holography an industry at last.

Although holographic imaging had provoked a storm of popular interest in the middle sixties, following the announcements by Leith and Upatnieks of off-axis transmission holography, holograms continued to be things that you had to go to darkened basements and museums to see—they were simply not bright enough to survive the glare of daylight. By 1972, McDonnell-Douglas Electronics had closed its pulsed-laser holography laboratory (which it had acquired with its purchase of Conductron, Inc., the Univ. Michigan spin-off company that had created so many impressive holograms for artists and industrial displays), and the rate of scientific publication in holography had fallen off to almost nothing. There was a major economic recession going on at the time, and peoples’ attentions turned to more immediately and economically promising technological challenges.

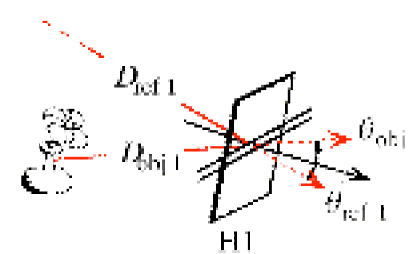
At Polaroid Corporation, a small laboratory had been established to study the applications of lasers to photographic problems, which also devoted a fraction of its efforts to display holography between manufacturing crises. In the course of some studies of full-aperture-transfer imaging and of bandwidth reduction concepts for electronic holography, a combination of the two ideas was found to hold promise for holographic television, and with a few changes it could instead produce <u>transmission</u> holograms that could be viewed with white light from ordinary sources, such as spotlights and the sun[1,2]. The key was the elimination of vertical parallax from the image, so that only side-to-side variations of the image’s perspective were presented—this was found to be sufficient for producing strong dimensionality in the image. White-light viewable <u>reflection</u> holograms had been known for several years, but the images they produced were dim, single-colored, and of low contrast (they will be described in subsequent chapters). The new white-light transmission holograms, or “rainbow holograms” as they came to be known, produced very bright and multi-colored images that could be shown in rooms filled with light. They were quickly adopted for artistic and commercial displays because of their vivid imagery.

However, individual glass-plate and film holograms were still expensive to produce, usually costing thousands of dollars each. But at RCA Corp., a scientist had proposed that the technique they had been using to produce LP records might be good enough to produce holograms cheaply, observing that LP record grooves were capable of diffracting light over fairly large angles if the music had high-frequency components[3]. The new process involved producing a surface-relief or undulating-surface grating, electroforming a hard-metal copy of the relief pattern, and using it to emboss or cast a replica surface on a sheet of transparent plastic, which was subsequently mirrorized so that it could be attached to a surface with adhesive (the process is described in more detail in the following chapter). This brought the cost of display holograms down to under a penny per square inch, cheap enough to be given away on magazine covers as attention getters, and eventually on credit cards as counterfeiting deterrents. Over the years, these “silvery blob” embossed holograms have become a standard part of many printers’ high-tech repertoire, and new variations are being developed constantly.

This chapter will describe the basic concepts of white-light transmission “rainbow” holography, and the next will pick up on some of the topics relevant to the state of the art in multi-color and embossed holograms. As we will see, the simplification in the viewing of rainbow holograms is won at the cost of some mathematical complexity in planning and making them. In particular, the details of *astigmatic* imaging will have to be taken into some account. We will first look at the process in the “forward” direction, from mastering to transferring to viewing. However, because of limitations in the viewer’s distance that have to be anticipated, we will find that it is more often necessary to work “backwards,” starting with the viewer’s intended location, which specifies first the transfer geometry, and then the mastering geometry.

Overview of the process

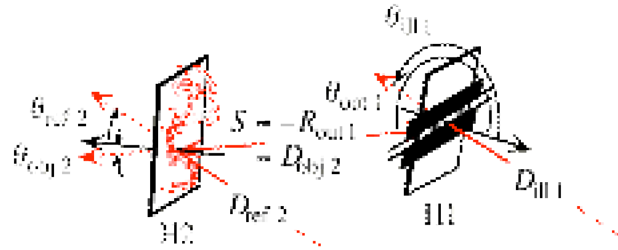
<u>Mastering:</u>



The process starts by recording a master hologram, or H1, although at a distance from the object that is usually quite a bit larger than used for full-aperture transfers. We will see that the object-to-H1 spacing, D_{obj1}, will eventually determine the optimum viewing distance, D_{view}, along with all the reference and projection beam distances, and will have to be carefully reckoned. For now, let’s assume that D_{obj1} is something handy, such as 300 mm (12"). As before, we can imagine that each small area of the plate, perhaps a half-millimeter on a side, records a unique perspective view of the scene corresponding to its location, from up to down and side to side.

Transferring:

Again, the H1 is illuminated in phase conjugation (or at least approximate phase conjugation) by bringing an illumination beam (sometimes called the "projection beam") through its back surface in a direction opposite to that of the reference beam. The convergence of the projection can also match the divergence of the reference beam—they are both typically collimated—or not. The transfer hologram, or H2, is now placed so as to straddle the projected real image (which is pseudoscopic), making the maximum image depth as small as is practical (as a rule). A reference beam is introduced at an angle, θ_{ref2}, usually from below, and from a distance, D_{ref2} , that is as large as the table permits (if a second collimator is not available).

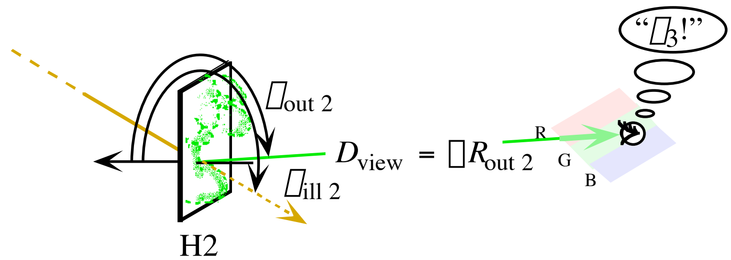


H1 as a line array of projectors:

We wish to project a continuum of images differing only in right-to-left perspective content. Recall that each area of the hologram can project only the perspective view that it recorded. Thus, by illuminating only a narrow horizontal stripe of the H1, we can eliminate the up-to-down variations of perspective within the projected real image. The choice of the slit's width is determined by practical considerations mentioned below, and the choice of optics to "feed" the slit without wasting light is also a topic for later discussion. But for now, we can imagine that we simply mask off most of the plate, perhaps with black photo tape applied directly to the back of the H1 (the front surface is typically index-matched to a clear-glass plateholder).

viewing of the H2:

The H2 may now be illuminated from above and behind with a monochromatic point source at the same wavelength with which it was recorded. The illumination is in the direction opposite to the reference beam, and the source distance is as large as possible so as to come as close to phase-conjugate illumination as possible. We can consider the H2's output in either of two ways: as an image of the real image projected by the H1, and as an image of the slit on the H1. Each point of view yields its own insights into the imaging process.



The H2 produces a pseudoscopic image of whatever its object exposure had been, which was itself a pseudoscopic image of the original object. "Two pseudos make an ortho," as we have seen before, so that a right-reading image is the final viewing result. It is visible from the direction of the image of the H1 slit, as before, but now we have to consider that slit image in more detail.

The real or aerial image of the H1 slit is formed at a fairly large distance from the H2, and its location is fairly sensitive to the exactness of the phase conjugation of the illumination relative to the reference beam. Typically, for the longest beam lengths available on practical tables, the slit image is about 1.5X as far from the H2 as the H1 was during the exposure. This departure from perfection also means that the slit image will suffer from astigmatism, with consequences that we will explore shortly.

perfect case: perfect conjugation

If the illumination is the perfect conjugate of the H2's reference beam (which usually means that the reference beam was converged with a large lens), the output slit will be located exactly as far from the H2 as the H1 had been. If the hologram is illuminated with a monochromatic point source of the same wavelength as the exposing laser, then the real image of the slit will be found exactly where the slit had been. All of the light diffracted by the H2 will focus through this slit image, and the viewer's eyes will have to be positioned accurately in that location. If the viewer moves up or down from there, the H2 abruptly turns dark! Moving side to side captures the light projected by various areas of the H1, which present images differing in horizontal perspective. This provides the difference between the right and left eyes' views, and motion parallax as the viewer moves from side to side.

If the wavelength/color of the light source is changed, the location of the slit real image changes in both angle and distance. It moves upwards for redder light, and downwards for bluer light, as was true for diffraction gratings. The redder image is also focused closer than the bluer image, as was true for Fresnel zone plates. Thus if the viewer moves up and down, instead of seeing different perspective views, i.e. different amounts of “look over” and “look under,” she/he sees the same image but in different monochromatic hues.

Because of the limited change of wavelength in going from deep red to deep blue viewing, the range of output angle is only about 15°, which means that the “window” for seeing anything is somewhat limited in height. The viewer must also be at roughly the intended distance to see the entire image in a single color, such as “green.” Moving too far backwards produces an image that is red at the top and blue at the bottom, while moving too near the hologram produces blue at the top and red at the bottom!

Let’s go through the numbers for a simple “ideal” phase conjugation case. Assume that we locate the object 600 mm in front of the H1 “master” plate. The reference beam will be a collimated beam, and the laser wavelength will be 532 nm (a doubled-YAG laser).

Upon back-illumination of the H1 with a collimated projection beam, the pseudoscopic real image of the object will be formed at unity magnification exactly 600 mm in front of the H1. This is where the H2 will be placed, so that it cuts the depth of the image roughly in two (the hologram plane “straddles” the image space). A slit is placed horizontally across the H1, blocking projection of the up-to-down variations of the views (the key step of the “rainbow” process). A collimated reference beam is used now for the H2, also of λ=532 nm, to expose the final “transfer” hologram, and the beam is arranged so that it comes up from below the H1 at 45°, anticipating the eventual illumination direction.

After careful processing, the hologram is held vertical, and illuminated from 45° above and behind with a collimated white light beam (such as sunlight). Considering only the 532 nm green light component of the sunlight spectrum for the moment, we find that the image of the H1 slit is formed directly in front of the hologram, at a distance of 600 mm. An eye placed there, anywhere along the width of the slit image, sees a undistorted unity-magnified image of the object floating within the H2 as if it were a window frame. Now, considering the 633 nm red component of the white light, the redder light is rotated more radically, and forms a slit image above the green-light slit image, and somewhat closer to the hologram. An eye placed at that new image location will see the same image as before, but in bright red light instead of green. The tonality and perspective will be the same—only the overall color will have changed. If the eye moves between these two locations, it will see the same image in a continuously changing spectral color, from green to yellow to orange to red. Contrariwise, if the eye moves downward, it will see the image in colors from green to cyan (blue-green) to blue to violet. It is the purity of these spectral colors that gave “rainbow” holograms their name.

As the eye moves from side to side within a single color zone, it picks up the images first captured by the corresponding regions of the master hologram, the H1. Eventually, the viewing zone “runs out of H1” and the image goes dark on the extreme right and left sides. Thus a viewing window is established that has its width determined by the width of the H1, and its height determined by the amount of spectral dispersion (typically about 15°).

<u>imperfect case: biggest problem</u>
Most holographers have either no collimators or just one, because they are so expensive, so that perfect conjugation isn’t available in either the transfer or the viewing stages, or both. This brings us to the <u>practical</u> side of rainbow holography, where our shop-math formulas help us place the image where we want it to be, regardless of the limitations of our equipment.

Let’s assume that we have no collimators at all, so that we have to use diverging beams for reference #1, projection, reference #2, and illumination. The price we will have to pay is that the object will have to be smaller than the image we want to produce, and it will be closer to the H1 than the viewing distance. In addition, there will be some distortions of the image that we will have to live with, or partially compensate by pre-distorting the object in a complementary way. Let’s assume for simplicity that all our beam-throws (wavefront radii) are the same, being 3 meters. The viewing distance will be 0.5 meter.

Other complications will also arise. First, the wavelength we shoot in will be different from our eventual viewing goal. The He-Ne laser wavelength is 633 nm (red), whereas our “target” viewing wavelength will be 550 nm (yellowish-green). Shrinkage of the emulsion layers will also be a concern, but for now we will assume that we “split the angle” between the object and reference beams for the H1, so that the fringes are vertical and thus unaffected in angle by shrinkage, and that we already know that to get a perpendicularly-exiting green beam from the H2, the red object beam has to have an angle of 7° to the perpendicular (see below).

Mathematics of WLT holograms: backwards analysis

With all these points in mind, we are ready to start designing the exposure setup for creating a rainbow hologram that fills a certain prescription. We start by considering the setup for the H2. From here on, we will consider the radii of the relevant wavefronts, rather than the distances to their sources:

H2 “transfer hologram” optics

The prescription above translates into illumination conditions of θ_{illum2} = +225°, R_{illum2} = +5000 mm, and viewing conditions of θ_{out} = 180°, $R_{out,vertical}$ = –500 mm, λ_3 = 550 nm.

These conditions require a little more explanation! The output angle is 180°, or perpendicular to the hologram, which is the usual case for holograms that are meant to hang vertically, whether by wires or in a frame. It is easy to check, because the viewer will see her eyes reflected in the center of the hologram if the angle is right. The viewing distance is chosen arbitrarily, but generally depends on the size of the hologram—the bigger the hologram, the larger the viewing distance! People are used to looking a television at a distance such that the screen subtends an angle that is about one fist wide, held at arm’s length (try it! You aren’t getting your money’s worth at a movie if it isn’t at least <u>three</u> fists wide!). So, a 4”x5” hologram is typically viewed at about half a meter (a bent arm’s length) and an 8”x10” at a full meter, and so on. The viewing distance is represented by a negative curvature of the output wavefront, because the hologram is creating waves that are meant to converge to the intended location of the eyes. What is somewhat subtle is that it is the <u>vertical</u> convergence that matters, so that the same color reaches the eye from the top and bottom of the hologram. Because of imperfect conjugation, the horizontal and vertical foci will be at noticeably different distances (the horizontal will be further away), and we have to make sure we use the appropriate equations to calculate the two distances. Finally, the choice of wavelength is also arbitrary—green (550 nm) simply defines the center of the viewing window for convenience. If we are making multi-color holograms, we will choose two or three other wavelengths for our calculations instead.

<u>getting the angles right:</u>

The first equation we need to deal with is the grating equation or “sine-theta equation” from Chapter X. We present it in symmetrical form to emphasize that the calculations proceed in both directions:

$$\frac{\sin\theta_{out2,\,m} - \sin\theta_{ill2}}{\lambda_3} = m\,\frac{\sin\theta_{obj2} - \sin\theta_{ref2}}{\lambda_2}, \qquad m = \pm 1\,. \tag{1}$$

Inserting the values for the variables from the example, we find that we have an arbitrary choice of pairs of reference and object beam angle that will give the same spatial frequency at the center of the hologram. For example, 56° and 0°, or 45° and -5°.

$$\frac{\sin 180° - \sin 225°}{550\text{ nm}} = -1\,\frac{\sin\theta_{obj2} - \sin\theta_{ref2}}{633\text{ nm}}\,. \tag{2}$$

The preferred choice is determined by a factor that we haven’t considered so far: that the hologram fringes form a “venetian-blind-like” structure in the emulsion that behaves like an array of tiny mirrors. Their angle has to be correct for the hologram to give maximum brightness when it is vertical, a phenomenon we will call “Bragg selection effects.” The result depends on how much the emulsion shrinks during processing, and by how much its refractive index changes. Typically, for silver halide holograms, the emulsion shrinks by 7% and the refractive index drops from 1.64 to 1.59. The calculations will be outlined in an appendix about the TK-Solver model called “tshrink.” The result in this case is that the object beam should have an angle of –5° and the reference beam an angle of 46.6°.

$$\theta_{obj2} = -5°, \quad \theta_{ref2} = 46.6°\,. \tag{3}$$

Note that the reference beam should come up to the H2 plate from “below” so that it can be illuminated from behind and above. This angle is usually difficult to arrange (unless you have a hole in the table, or some mirrors cleverly arranged!), so the H2 is usually turned on its side so that the reference beam can travel horizontally across the table.

<u>getting the distances right:</u>

The key distance to consider here is the viewing distance, D_{view}, at the intended wavelength, λ_3, and viewing angle (typically 0°, but the dependence on angle is quite small). And the key realization is that it is the <u>vertical</u> or color focus that is relevant—this is the peculiar astigmatic focus that we discovered is an effect in off-axis holograms, recall. The reason that this is focus that matters is that we wish to see the same color, green in this case, coming from the top and bottom of the hologram. That is, we want to find the point where the green rays from the top, center, and bottom all cross. An eye placed there will see the entire hologram surface light up in bright green light! The other focus, where the green rays from the right, center, and left all cross determines where one can view the exact perspective captured by a region of the slit on the H1 master hologram (which doesn’t matter at this point).

The vertical focus is determined by the “cosine-squared” equation, which again we show in symmetrical form:

$$\frac{1}{\lambda_3}\left(\frac{\cos^2\theta_{\mathrm{out2},m}}{R_{\mathrm{out2},m,\mathrm{V}}} - \frac{\cos^2\theta_{\mathrm{ill2}}}{R_{\mathrm{ill2}}}\right) = m\frac{1}{\lambda_2}\left(\frac{\cos^2\theta_{\mathrm{obj2}}}{R_{\mathrm{obj2}}} - \frac{\cos^2\theta_{\mathrm{ref2}}}{R_{\mathrm{ref2}}}\right) . \tag{4}$$

Inserting the values for the variables that were discussed above, we find that the object beam must be a diverging beam with a positive radius of curvature of 391 mm. This means that the slit of the H1 must be 391 mm away from the H2, so we have determined the H1–H2 separation, usually called "*S*."

$$\frac{1}{550}\left(\frac{\cos^2 180°}{-500\text{ mm}} - \frac{\cos^2 225°}{5000\text{ mm}}\right) = -1\frac{1}{633}\left(\frac{\cos^2(-5°)}{R_{\mathrm{obj2}}} - \frac{\cos^2 46.6°}{5000\text{ mm}}\right), \tag{5}$$

$$S = R_{\mathrm{obj2}} = 391\text{ mm} . \tag{6}$$

H1 "master hologram" optics

Now we are faced with the challenge of creating a master hologram, or H1, that will project a real image at the proper angle and distance (*S*) so as to straddle the H2 plane, or at least to put the image where we want it in front of or behind the transfer hologram surface.

The angles are fairly straightforward, given a couple of practical considerations. First, the projection beam for the H1 should be parallel to the reference beam for the H2, just for convenience in getting them both as long as possible within the constraints of the table. This gives the relation:

$$\left(\theta_{\mathrm{obj1}} - \theta_{\mathrm{ref\,1}}\right) = -\left(\theta_{\mathrm{obj\,2}} - \theta_{\mathrm{ref\,2}}\right) . \tag{7}$$

Second, the object and reference beams should come in at equal but opposite angles to the perpendicular to the H1. This makes the resulting interference fringes perpendicular to the surface of the emulsion, so that there is no astigmatism in the focus of the image, and the fringe tip angle is insensitive to shrinkage of the emulsion—this widens our choices of processing chemistry considerably. It is also easy to check when the hologram perpendicular is at the right angle—the hologram will reflect the reference beam onto the object!

Assuming that the H1 and H2 are exposed at the same wavelength, there is no adjustment for wavelength change effects, and the exposing angles are simply:

$$\begin{aligned} 2\left(\theta_{\mathrm{obj1}} = -\theta_{\mathrm{ref\,1}}\right) &= -(-5° - 46°) \\ \theta_{\mathrm{obj1}} &= +21.5° \\ \theta_{\mathrm{ref\,1}} &= -21.5° \end{aligned} \tag{8}$$

The exposure distance, object-to-H1, is only slightly more difficult to find. The output wavefront must have a radius of "negative *S*" in order to converge at the required H2 location. The relevant axis of focus is now the horizontal or parallax focus, because the image focus is determined by the distance at which rays from the right, center and left areas of the H1 slit overlap. This we need the simpler "one-over-*R*" equation.

$$\frac{1}{\lambda_2}\left(\frac{1}{R_{\mathrm{out1},m,\mathrm{H}}} - \frac{1}{R_{\mathrm{ill1}}}\right) = m\frac{1}{\lambda_1}\left(\frac{1}{R_{\mathrm{obj1}}} - \frac{1}{R_{\mathrm{ref1}}}\right) \quad , \quad m = -1 . \tag{9}$$

Substituting the values of the variables involved gives:

$$\begin{gathered} \frac{1}{633}\left(\frac{1}{-391} - \frac{1}{5000}\right) = -1\frac{1}{633}\left(\frac{1}{R_{\mathrm{obj1}}} - \frac{1}{5000}\right) \quad , \\ R_{\mathrm{obj1}} = 338\text{ mm} . \end{gathered} \tag{10}$$

That S is greater than D_{obj} means that the image will be magnified side-to-side by the same ratio, and magnified in depth by the square of that (which can become a lot!). Clever holographers often pre-distort their objects to compensate, so that intended spheres become small, shallow, dish-shaped objects. Previsualization of a hologram in the face of all these distortions becomes quite a challenge. Some folks use wire frames to help compose their scenes, or distorted checkerboards.

Other effects of imperfect conjugates:

Not having enough collimators causes other problems, too. These are primarily apparent in the image projected by the H1, where the distances from the hologram are large, but can be seen in the way the H2 plays back too. The generic name for the effects of imperfect conjugates is “optical aberrations.” L. Seidel identified and named these for conventional optical systems back in 1856, and we can adapt them for holographic discussions too. The five Seidel or primary aberrations are:

spherical aberration: a lens with spherical surfaces doesn’t usually produce a perfectly spherical wavefront—instead, it curves inward more sharply when measured further from the center.

astigmatism: light passing at an angle through a lens generally has different curvatures in the direction toward the central axis (defining the *sagittal* focus) and perpendicular to it (defining the *tangential* focus).

coma: even if astigmatism is cured, as the lens diameter increases the light will focus at different angles and distances, producing a diffuse comet-light tail around the sharp centrally-formed point.

curvature of field: the image of a flat surface (or a constellation of stars) formed by a lens is only approximately flat—the surface of best focus is usually cupped slightly toward the lens.

distortion: the image of a checkerboard is usually bowed inward or outward at the edges, termed “pincushion” and “pillow” distortion respectively. This arises from the output angle of the lens being non-linearly related to the input angle—its effects in holography are not discussed here.

When projecting an image with an H1, aberrations arise when an imperfect conjugate wave is used for illumination. The biggest problems are caused by spherical aberration, which causes the hologram image of a flat surface to curve away from the hologram plane, being closest directly in front of the viewer, and to “roll” as the viewer moves from side to side. Coma also arises, which causes the image of a point to move up and down as the viewer moves from side to side. The “trail” of a point in the hologram plane can trace out some strange shapes, instead of the straight horizontal line predicted by simple theory, which can cause eyestrain in extreme cases. The particular mix of aberrations found reflects the holographer’s choices of equipment, and sometimes you can identify a particular holographer’s work just from the shape of the “trails” of bright points in the image!

Slit width questions

One of the perennial questions for rainbow holographers is “how wide a slit should I use?” The best answer can vary between 0.5 mm and 25 mm, and depends very much on the nature of the image. A thin (0.5–2 mm) slit gives very sharp images over great depths (perhaps 150 mm in front of and behind the hologram), but with high speckle contrast. As the slit is widened, the speckle slowly decreases in contrast so as to become nearly invisible (8–25 mm), but the image starts to be blurred at shallower depths. Only a few experiments will provide a useful answer, and will typically require a compromise between depth and speckle[4].

As a practical matter, as much of the H1 illumination as possible is fed to the slit area by using cylindrical lenses to spread the beam upstream of the H1. If more beam width control is needed, crossed cylindrical lenses of very different focal lengths are used, often with a collimating lens to control the spreading of the beam. Cylindrical lenses can be expensive, but a test tube full of mineral oil, or a carefully-chosen section of polished glass rod, can usually suffice.

Limitations due to horizontal-parallax-only imaging

Rainbow holograms are white-light viewable because they sacrifice one axis of parallax—they produce “horizontal parallax only” images (HPO images). Conceptually, we can say that the entropy of the hologram (its information content) has been reduced to match the reduced entropy of the light source (its temporal coherence). However, there are other techniques for producing HPO images, such as the use of lenticules (small vertical cylinders embossed/cast into the surface of a plastic sheet), as seen on 3D postcards. All HPO images share certain limitations or optical effects that should not be attributed to holograms in particular:

inherent astigmatism

In a horizontal plane, the rays from an image point fan out from the point’s apparent location behind the hologram surface, a central principle of stereoscopy. But the rays fanning out in a vertical plane always have their common center on the hologram surface—a point on the horizontal “track” of the image point. The result is a stigmatic ray bundle, or a wavefront that has different curvatures in the horizontal and vertical directions—with a difference that increases as the image location moves further from the hologram surface.

depth of field:

The human eye can tolerate only a limited amount of astigmatism before eyestrain results (the eye continually refocuses to try to sharpen the image). Optometrists usually allow “one quarter diopter” of astigmatism before changing a prescription to correct it. In our terms, that translates to

$$\begin{aligned} \Delta_{\mathrm{astig1}} &= \frac{1}{D_{\mathrm{near}}} - \frac{1}{D_{\mathrm{holo}}} \le 0.25 \text{ meters}^{-1} \\ \Delta_{\mathrm{astig2}} &= \frac{1}{D_{\mathrm{holo}}} - \frac{1}{D_{\mathrm{far}}} \le 0.25 \text{ meters}^{-1} \end{aligned} \tag{11}$$

That is for a hologram viewed from 500 mm away, the image point can be 56 mm in front of the hologram, or 71 mm behind the hologram, before viewing becomes stressful. Art holographers deliberately violate this limit as a matter of course, assuming that nobody will be looking at any one image for very long. But we should also recall that someone with 1/4-diopter of uncorrected astigmatism will be able to tolerate more depth on one side of the hologram, and less on the other.

viewer distance limitations

The same astigmatism effect produces a distortion of the image when the viewer is not at the correct distance (defined now as the distance to the horizontal focus of the H1 image). The image of a spherical object, or ball, floating in front of the hologram will appear squashed, or shrunken up-to-down, as the viewer moves further than the intended distance, and stretches up-to-down as the viewer moves closer. Fortunately, the human eye is quite tolerant of height-to-width distortions, so that a useful range of viewing distances can be accepted.

spectrum tip

For most people, the correct viewing distance is the one at which the image appears in a single color from top to bottom. That is, the one formed by the vertical focus of the H1 image). This is usually calculated for the middle-green wavelength, 550 nm. If the eye moves upward, a yellower, then redder image is seen. However, we should note that the optimum viewing distance also shrinks considerably, so that the surface of optimum viewing turns out to be a plane that is tipped forward. The angle of tip is what we identified earlier as the “achromatic angle,” or α., and is somewhat greater than the angle of the illumination beam. In detail, $\tan\alpha = \sin\theta_{\mathrm{ill}}$.

Conclusions

The principal advantages of rainbow holograms is that their images are sharp and deep when viewed with commonly available light sources, and they can be very bright. No light is wasted by narrow-band filtering, and the light that is diffracted (the image light) is sent into a beam that is quite narrow, vertically (only about 15° high). Thus relatively weak unfiltered spotlights can be used to illuminate them—even flashlights and candles work well! Unfortunately, they are a little difficult to produce, especially if the highest levels of quality are desired. The exposure system must be carefully designed in order to produce the desired effect under the specified illumination and viewing conditions.

Fortunately, the tolerance for error is high enough to allow a wide range of viewing conditions to produce acceptable images, and mass-produced rainbow holograms have become very popular. The next chapter will consider some of the issues of making practical rainbow holograms, including multi-color holograms and embossed holograms, all based on the same principles that are outlined here.

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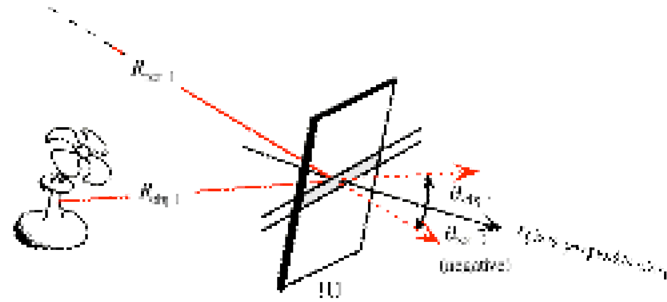
WHITE-LIGHT TRANSMISSION "RAINBOW" MATHEMATICS

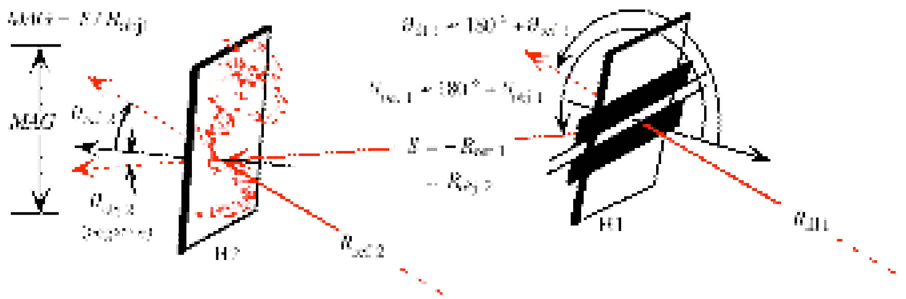
1) mastering @ λ_1



$$\frac{1}{\lambda_2}\left(\sin(\theta_{out\,1}) - \sin(\theta_{ill\,1})\right) = (-1)\,\frac{1}{\lambda_1}\left(\sin(\theta_{obj\,1}) - \sin(\theta_{ref\,1})\right)$$

$$\frac{1}{\lambda_2}\left(\frac{1}{-S} - \frac{1}{R_{ill\,1}}\right) = (-1)\,\frac{1}{\lambda_1}\left(\frac{1}{R_{obj\,1}} - \frac{1}{R_{ref\,1}}\right)$$

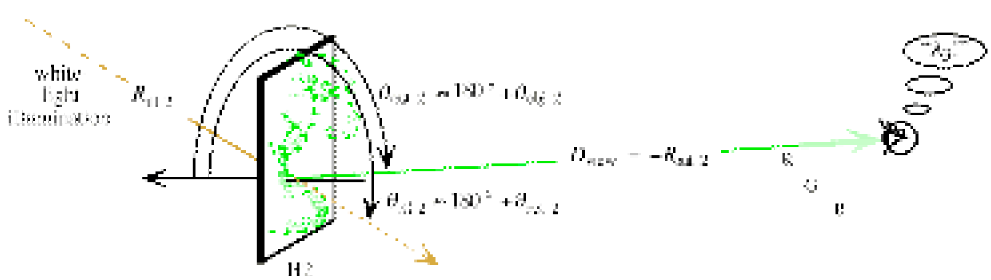
2) transferring @ λ_2



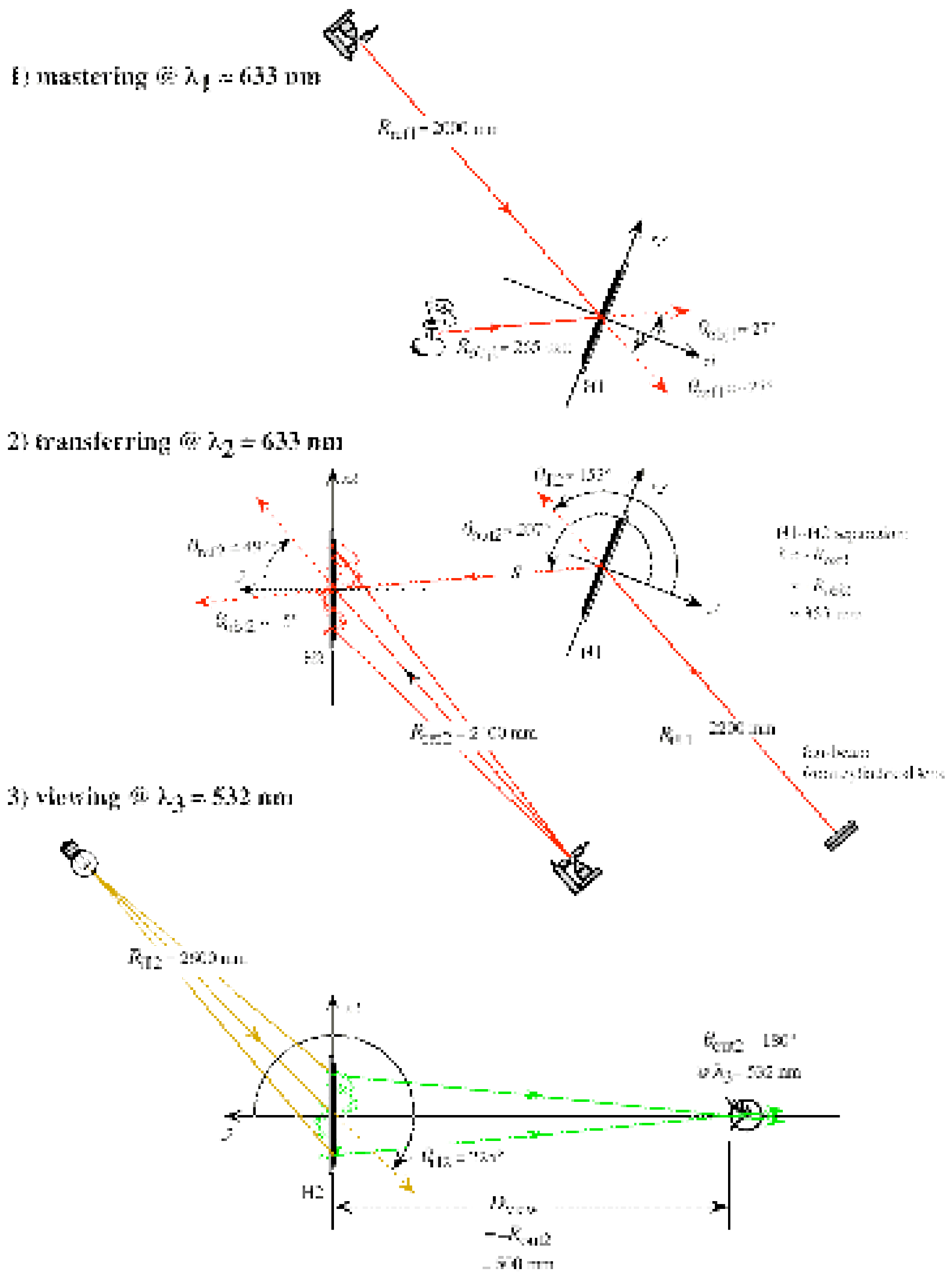
$$\frac{1}{\lambda_3}\left(\sin(\theta_{out\,2}) - \sin(\theta_{ill\,2})\right) = (-1)\,\frac{1}{\lambda_2}\left(\sin(\theta_{obj\,2}) - \sin(\theta_{ref\,2})\right)$$

$$\frac{1}{\lambda_3}\left(\frac{\cos^2(\theta_{out\,2})}{-D_{view}} - \frac{\cos^2(\theta_{ill\,2})}{R_{ill\,2}}\right) = (-1)\,\frac{1}{\lambda_2}\left(\frac{\cos^2(\theta_{obj\,2})}{S} - \frac{\cos^2(\theta_{ref\,2})}{R_{ref\,2}}\right)$$

3) viewing @ λ_3



WHITE-LIGHT TRANSMISSION "RAINBOW" MATHEMATICS — an example
(the numbers are different than for the chapter text)



n.b. not to scale