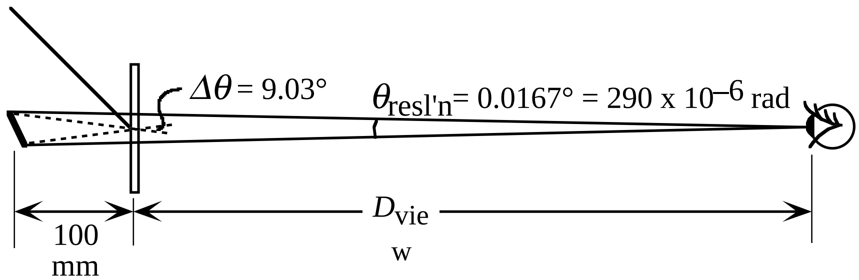
MAS.450/854 Holographic Imaging
SOLUTIONS: Prob. Set #4: Full-Aperture Transfers & White-Light Transmission Holograms

U1) An image of a point is 100 mm <u>behind</u> a white-light illuminated full-aperture transfer hologram illuminated with white-light at 60°. How far from the hologram must a viewer be so that the separation between the images formed in 500 nm and 600 nm light is invisible (assume visual resolution of one minute of arc).

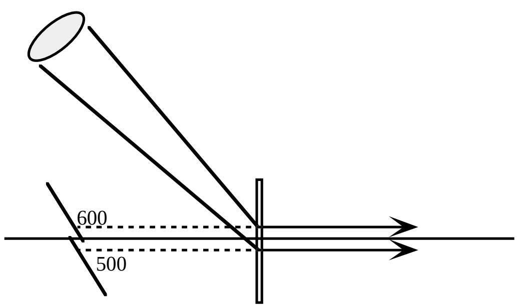
(8 pts)



$\theta_{ill} = -60°$, and $\theta_{out,\,550\,nm} = 0°$; $\theta_{out,\,600\,nm} - \theta_{out,\,500\,nm} = \Delta\theta = 4.52° - (-4.52°) = 9.03°$ (0.16 rad)

$$\tan 0.0167° = \frac{100\text{ mm} \times \tan 9.03° = 15.9\text{ mm}}{D_{view} + 100\text{ mm}} \qquad D_{view} = 54.4\text{ meters}$$

What diameter of illuminating light source 5 meters away at 60° will blur the same image points together?



$$f_{550nm} = \frac{0 - \sin(-60°)}{550 \times 10^{-6}\text{mm}} = 1575\text{ cy / mm}$$

$$\sin\theta_{out} = m\lambda f + \sin\theta_{ill}$$

$$0 = 1 \cdot 600 \times 10^{-6} \cdot 1575 + \sin\theta_{ill,600}$$

$$\sin\theta_{ill,600} = -70.87°$$

$$\sin\theta_{ill,500} = -51.93°$$

$$\Delta\theta = 18.93°$$

$$\text{diameter} = R \cdot \tan\Delta\theta = 1.71\text{ meters}$$

alternatively, and more simply: $$h_{\text{source-blur}} = R_{out}\cos\theta_{ill}\left(\Delta\theta_{ill} = \frac{\Phi_{ill}}{R_{ill}}\right),$$

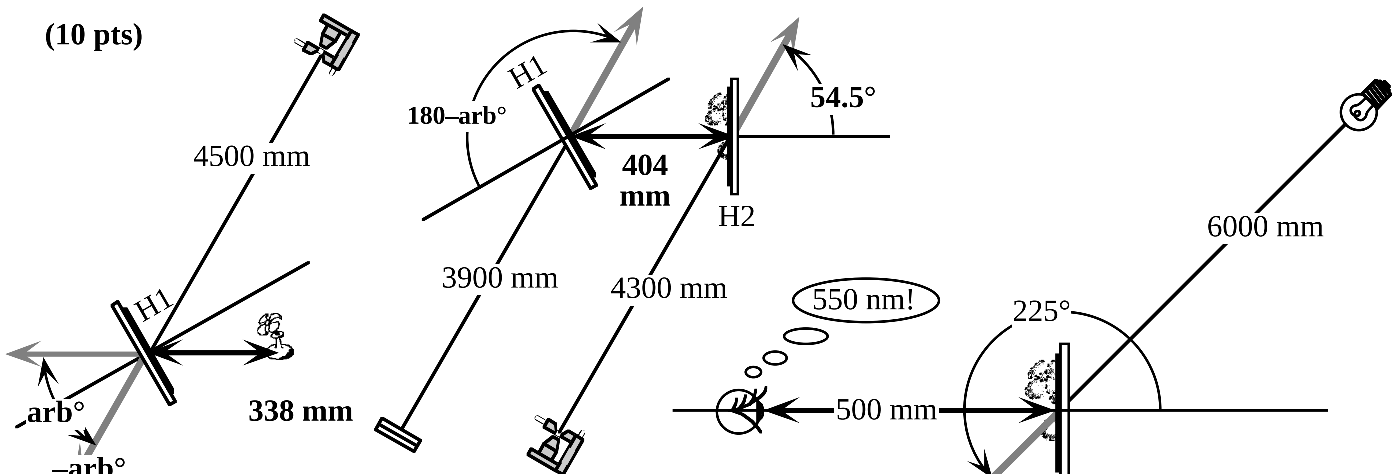
$$\Phi_{ill} = \frac{h_{\text{source-blur}}}{\cos\theta_{ill}}\frac{R_{ill}}{R_{out}}$$

$$= \frac{15.9}{\cos(60°)}\frac{5000}{100} = 1.59\text{ meters}$$

For every problem, especially on quizzes:
It is very important to give a fully labelled SKETCH, showing the numerical results you have obtained. Often you can salvage some partial credit if the sketch shows that you got nearly the right result, even if the numbers are strange!

U2) An 8”x10” rainbow hologram is designed to be illuminated from about five meters away at forty-five degrees above, and seen from straight ahead in 550 nm (“green”) light, per sketch C below. The longest beam throw available on the table is just over four meters, which limits the He-Ne laser reference and projection beams per sketches A and B. Determine the central object distance, the H1-H2 separation, and the H2 reference beam angle (using a thin hologram model); that is, complete the mastering and transfer setup designs, with diagrams showing **ALL** distances and angles.

(All exposures for both problems are made at λ = 633 nm)



$$\lambda_1 = \lambda_2 = 633\,\text{nm},\ \lambda_3 = 550\ \text{nm}$$

$$\theta_{obj1} = ?°,\ \theta_{ref1} = ?°,\ \theta_{out1} = ?°,\ \theta_{out1} = ?°,\ \theta_{ill1} = ?°$$

$$R_{obj1} = ?\,\text{mm},\ R_{ref1} = 4500\ \text{mm},\ R_{out1} = ?\,\text{mm},\ R_{ill1} = 3900\ \text{mm}$$

$$\theta_{obj2} = 0°,\ \theta_{ref2} = ?°,\ \theta_{out2} = 180°,\ \theta_{out2} = 180°,\ \theta_{ill2} = 225°$$

$$R_{obj2} = ?\,\text{mm},\ R_{ref2} = 4300\ \text{mm},\ R_{out2} = -500\ \text{mm},\ R_{ill2} = 6000\ \text{mm}$$

a) <u>angles for H2</u>:

$$\frac{\sin\theta_{out} - \sin\theta_{ill}}{\lambda_2} = m\,\frac{\sin\theta_{obj} - \sin\theta_{ref}}{\lambda_1}$$

$$\frac{\overset{0}{\sin(180°)} - \overset{-.707}{\sin(225°)}}{550} = (-1)\,\frac{\overset{0}{\sin(0)} - \sin\theta_{ref}}{633}$$

$$\sin\theta_{ref} = \frac{633}{550}(-0.707)\,;\quad \theta_{ref} = 54.5°$$

b) <u>angles for H1</u>:

We don't care what the reference and object angles are for H1, as long as they are equal and opposite (so that the fringes are perpendicular to the plate). We will call the chosen angle **arb°**. For approximate phase conjugation, the illumination angle must be equal to the reference beam angle plus 180°.

c) <u>distances for H2</u> (vertical or color focus):

$$\frac{\dfrac{\cos^2\theta_{out}}{R_{out}} - \dfrac{\cos^2\theta_{ill}}{R_{ill}}}{\lambda_2} = m\,\frac{\dfrac{\cos^2\theta_{obj}}{R_{obj}} - \dfrac{\cos^2\theta_{ref}}{R_{ref}}}{\lambda_1}$$

$$\frac{\dfrac{\overset{1}{\cos^2(180°)}}{-500} - \dfrac{\overset{0.5}{\cos^2(225°)}}{6000}}{550} = (-1)\,\frac{\dfrac{\overset{1}{\cos^2(0)}}{S} - \dfrac{\overset{0.26}{\cos^2(54.5°)}}{4300}}{633}$$

$$S = 404\ \text{mm}$$

d) <u>distances for the H1</u> (horizontal or parallax focus):

$$\frac{\dfrac{1}{R_{out}} - \dfrac{1}{R_{ill}}}{\lambda_2} = m\,\frac{\dfrac{1}{R_{obj}} - \dfrac{1}{R_{ref}}}{\lambda_1}$$

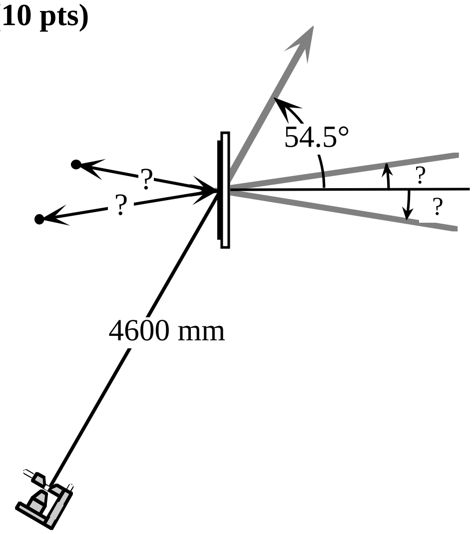
$$\frac{\dfrac{1}{-404} - \dfrac{1}{3900}}{633} = (-1)\,\frac{\dfrac{1}{D_{obj}} - \dfrac{1}{4500}}{633}$$

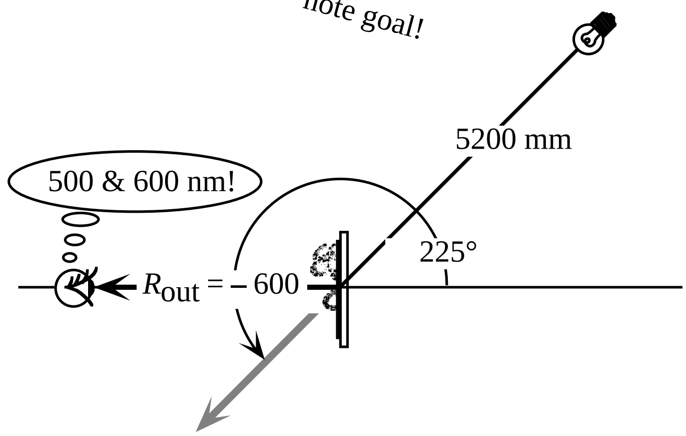
$$D_{obj} = 338\ \text{mm}$$

U3) Find the master slit locations needed to produce “blue-green” 500 nm and “orange” 600 nm images on-axis, per the sketch below. Find the angle of the line connecting the two slit locations.

<— note goal!

(10 pts)





angles first:

500 nm:

$$\frac{\sin 180^\circ - \sin\theta_{ill}}{\lambda_2} = m\,\frac{\sin\theta_{obj} - \sin\theta_{ref}}{\lambda_1}$$

$$\frac{\sin 180^\circ - \sin 225^\circ}{500} = (-1)\,\frac{\sin\theta_{obj,500} - \sin 54.5^\circ}{633}$$

$$\theta_{obj,500} = -4.65^\circ$$

600 nm:

$$\frac{\sin 180^\circ - \sin\theta_{ill}}{\lambda_2} = m\,\frac{\sin\theta_{obj} - \sin\theta_{ref}}{\lambda_1}$$

$$\frac{\sin 180^\circ - \sin 225^\circ}{600} = (-1)\,\frac{\sin\theta_{obj,600} - \sin 54.5^\circ}{633}$$

$$\theta_{obj,600} = 3.91^\circ$$

then: distances (vertical foci):

$$\frac{\frac{\cos^2\theta_{out}}{R_{out}} - \frac{\cos^2\theta_{ill}}{R_{ill}}}{\lambda_2} = m\,\frac{\frac{\cos^2\theta_{obj,500}}{R_{obj,500}} - \frac{\cos^2\theta_{ref}}{R_{ref}}}{\lambda_1}$$

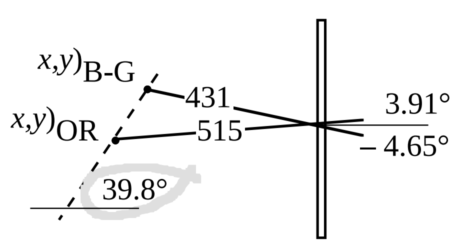
$$\frac{\frac{\cos^2 180^\circ}{-600} - \frac{\cos^2 225^\circ}{5200}}{500} = (-1)\,\frac{\frac{\cos^2(-4.65^\circ)}{R_{obj,500}} - \frac{\cos^2 54.5^\circ}{4600}}{633}$$

$$R_{obj,500} = 431\text{ mm}$$

$$\frac{\frac{\cos^2\theta_{out}}{R_{out}} - \frac{\cos^2\theta_{ill}}{R_{ill}}}{\lambda_2} = m\,\frac{\frac{\cos^2\theta_{obj,600}}{R_{obj,600}} - \frac{\cos^2\theta_{ref}}{R_{ref}}}{\lambda_1}$$

$$\frac{\frac{\cos^2 180^\circ}{-600} - \frac{\cos^2 225^\circ}{5200}}{600} = (-1)\,\frac{\frac{\cos^2(3.91^\circ)}{R_{obj,600}} - \frac{\cos^2 54.5^\circ}{4600}}{633}$$

$$R_{obj,500} = 515\text{ mm}$$



note: $\tan^{-1}(\sin 54.5^\circ) = 39.1^\circ$!