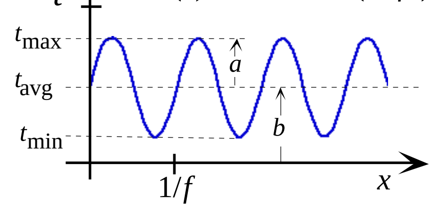
The Diffraction Efficiency of a Sinusoidal Transmission Grating:



$t(x) = b + a \sin(2\pi f x)$

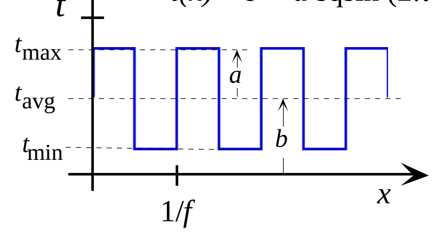
$DE_0 = b^2$

$DE_{+1} = DE_{-1} = \left(\frac{a}{2}\right)^2$

$DE_{m = \pm 2, \pm 3 \ldots} = 0$

$DE_{+1,\,\mathrm{MAX}} = 6.25\%$

The Diffraction Efficiency of a Square-Wave Transmission Grating:



$t(x) = b + a\ \mathrm{sqsin}(2\pi f x)$

$DE_0 = b^2$

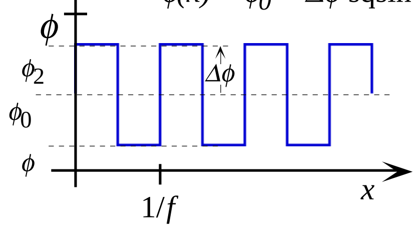
$DE_{+1} = DE_{-1} = \left(\frac{2a}{\pi}\right)^2$

$DE_{m = \mathrm{even}} = 0 \quad DE_{m = \mathrm{odd}} = \frac{1}{m^2}\, DE_{+1}$

$\sum_{m \neq 0} DE_m = a^2 = 25\%_{\max}$

$DE_{+1,\,\mathrm{MAX}} = 10.1\%$

The Diffraction Efficiency of a Square-Wave Phase Grating:



$\phi(x) = \phi_0 + \Delta\phi\ \mathrm{sqsin}(2\pi f x)$

$DE_0 = \cos^2 \Delta\phi$

$DE_{+1} = DE_{-1} = \left(\frac{2}{\pi}\sin\Delta\phi\right)^2$

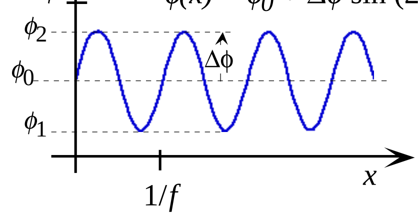
$DE_{m = \mathrm{even}} = 0 \quad DE_{m = \mathrm{odd}} = \frac{1}{m^2}\, DE_{+1}$

$\sum_{m \neq 0} DE_m = \sin^2 \Delta\phi = 100\%_{\max}$

$DE_{+1,\,\mathrm{MAX}} = 40.5\%$

$@\Delta\phi = \pi/2 = 90°$

The Diffraction Efficiency of a Sinusoidal Phase Grating:



$\phi(x) = \phi_0 + \Delta\phi \sin(2\pi f x)$

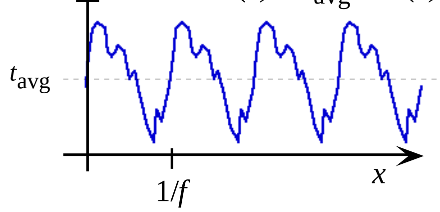
$DE_0 = J_0^2(\Delta\phi)$

$DE_{+1} = DE_{-1} = J_1^2(\Delta\phi)$

$DE_{+1,\,\mathrm{MAX}} = 33.8\%$

$@\Delta\phi = 1.86 = 0.59\pi = 107°$

The Diffraction Efficiency of a Generalized (complex transmittance) Grating:



$t(x) = t_{avg} + \delta t(x)$

$DE_0 = \left|t_{avg}\right|^2$

DE_{+1} = power spectrum component

$\sum_{m \neq 0} DE_m = \overline{|t(x)|^2} - DE_0 = \overline{|\delta t|^2}$

$\equiv \mathrm{var}\ t = \sigma_t^2$

$DE_{+1,\,\mathrm{MAX}} = 100\%$

note: These all assume that the grating is "optically thin." That is, that $Q = \frac{2\pi\lambda\,\mathrm{thickness}}{n\, d^2 \cos\theta_0} \ll 1$