MAS.450/856: Holographic Imaging
SOLUTIONS: Problem Set #5: Conformal Fringes & “Denisyuk” Holograms

Include **sketches** wherever possible to clarify your analyses or descriptions; they will often make a big difference!

1. Reflection Holography: An emulsion of refractive index 1.60 is exposed as shown in sketch 1. After processing, the emulsion has a refractive index of 1.50, and is found to have shrunk by 20%. Find the central wavelength and angle of the diffracted beam when the hologram is illuminated per sketch 2.

FIG 1) $\lambda_1 = 633$ nm
$n_1 = 1.60$

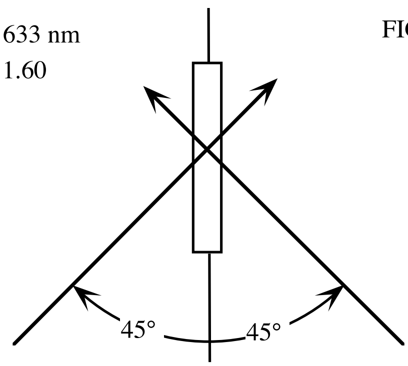
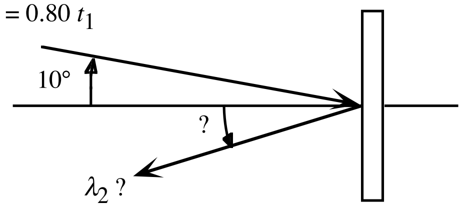
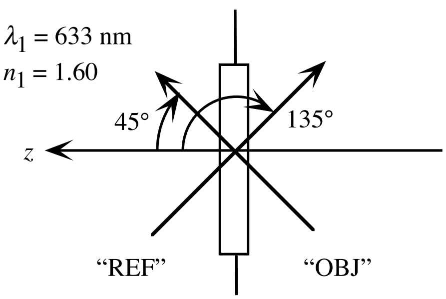


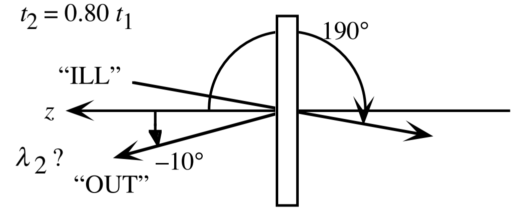
FIG 2) $n_2 = 1.50$
$t_2 = 0.80\ t_1$



(8 points)

There are two sources of color change: 1) the shrinkage effect, and 2) the <u>anti</u> blue-shift due to <u>reducing</u> the angle of illumination. The assignment of “object” and “reference” beams is arbitrary, but convenient when done this way:





the output angle follows from the mirror-like nature of holographic reflections, especially from conformal firnges

simply done, the wavelength is:

$$\frac{\lambda_2}{n_2 \cdot t_2 \cdot \cos\theta_{\text{ill}}^{\text{int}}} = \frac{\lambda_1}{n_1 \cdot t_1 \cdot \cos\theta_{\text{ref}}^{\text{int}}}$$

$$\lambda_2 = \frac{0.80\,t_1}{t_1}\,\frac{1.50}{1.60}\,\frac{\cos 153.8°}{\cos 186.6°}\,633 = 526 \text{ nm}$$

or 26.2°

or 6.6°

where:

$$\theta_{\text{ref}}^{\text{int}} = 153.8° = \sin^{-1}\left(\frac{\sin 135°}{1.60}\right)$$

$$\theta_{\text{ill}}^{\text{int}} = 186.6° = \sin^{-1}\left(\frac{\sin 190°}{1.50}\right)$$

note: the quadrants have to be kept track of “by hand”

or, more elaborately:
where Λ_1 and Λ_2 are the spacings of the reflective fringes

$$\frac{1}{\Lambda_1} = \frac{2}{\left(\frac{\lambda_1}{n_1}\right)}\sin\left(\frac{\theta_{\text{ref}}^{\text{int}} - \theta_{\text{obj}}^{\text{int}}}{2}\right)$$

$$= \frac{2}{\left(\frac{633\,\text{nm}}{1.60}\right)}\sin\left(\frac{153.8° - 26.2°}{2}\right)$$

$$= \frac{1}{220\,\text{nm}}$$

$$\Lambda_2 = 0.80\ \Lambda_1 = 176 \text{ nm}$$

$$\frac{1}{\Lambda_2} = \frac{2}{\left(\frac{\lambda_2}{n_2}\right)}\sin\left(\frac{\theta_{\text{ill}}^{\text{int}} - \theta_{\text{out}}^{\text{int}}}{2}\right)$$

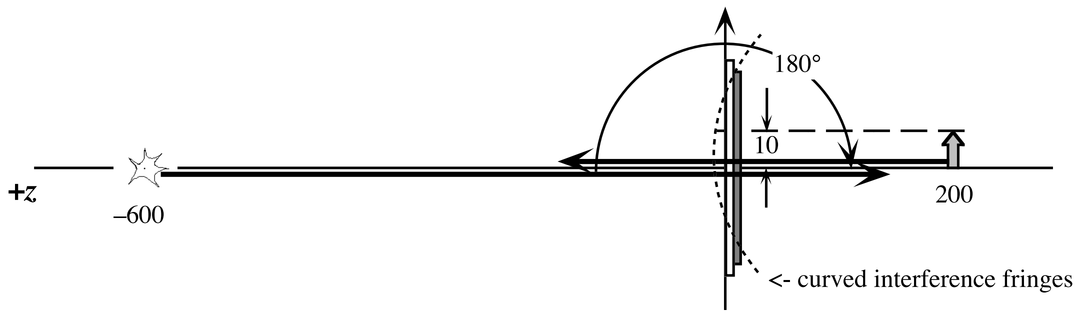
$$\frac{1}{176\text{ nm}} = \frac{2}{\left(\frac{\lambda_2}{1.50}\right)}\sin\left(\frac{186.6° - (-6.6°)}{2}\right)$$

$$\lambda_2 = 526\,\text{nm}$$

2. A holographic plate is exposed with He-Ne laser light in the setup sketched below (distances are in millimeters). After processing, it is found to have shrunk by 5%. It is then viewed with an on-axis white-light point source two meters away. The average refractive index of the hologram is 1.0 during exposure and viewing.

a) Find the <u>location</u> and magnification of the image if the hologram is illuminated from the glass side.

b) Find the <u>location</u> and magnification of the image if the hologram is illuminated from the emulsion side.



(6 points)

a) $$\frac{\lambda_2}{n_2 \cdot t_2} = \frac{\lambda_1}{n_1 \cdot t_1}$$

$$\lambda_2 = \frac{0.95\, t_1}{t_1}\,\frac{1.0}{1.0}\, 633 = 601.4 \text{ nm}$$

recall that angles are defined by reference to the z-axis in the direction of the OBJECT beam

$$\sin\theta_{out} = m\frac{\lambda_2}{\lambda_1}\left(\sin\theta_{obj} - \sin\theta_{ref}\right) + \sin\theta_{ill}$$

$$= (1)(0.95)\left(\sin 0° - \sin 180°\right) + \sin 180°$$

$$= 0$$

$$\theta_{out} = 0° \text{ or } 180°$$

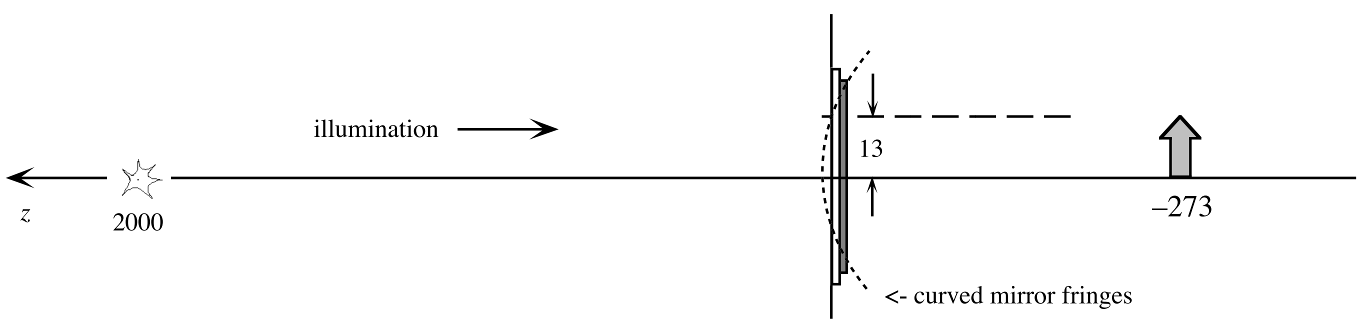
$$\frac{1}{R_{out}} = m\frac{\lambda_2}{\lambda_1}\left(\frac{1}{R_{obj}} - \frac{1}{R_{ref}}\right) + \frac{1}{R_{ill}}$$

$$= (1)(0.95)\left(\frac{1}{200} - \frac{1}{600}\right) + \frac{1}{2000}$$

$R_{out} = 273\text{mm}$ $\quad (x,z) = (\ 0, -273\ )$ mm (location!)

$$\text{MAG}_{lat} = (1)\left(\frac{273}{200}\right)(0.95) = 1.30$$

$$\text{MAG}_{long} = \left(\frac{1}{0.95}\right)(1.30\ )^2 = 1.77$$



(4 points)

b)

$$\sin\theta_{out} = (-1)(0.95)(\sin 0° - \sin 180°) + \sin 0°$$
$$= 0$$
$$\theta_{out} = 0° \quad \text{or } 180°$$

$$\frac{1}{R_{out}} = (-1)(0.95)\left(\frac{1}{200} - \frac{1}{600}\right) + \frac{1}{2000}$$

$$R_{out} = -375 \text{ mm}$$ $(x,z) = ($ 0, 375 $)$ mm (location!)

$$M_{lat} = (-1)\left(\frac{-375}{200}\right)(0.95) = 1.78$$

$$M_{long} = \left(\frac{-1}{0.95}\right)(1.78)^2 = -3.34$$