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# Computational Imaging:

## A Survey of Medical and Scientific Applications

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Douglas Lanman\*

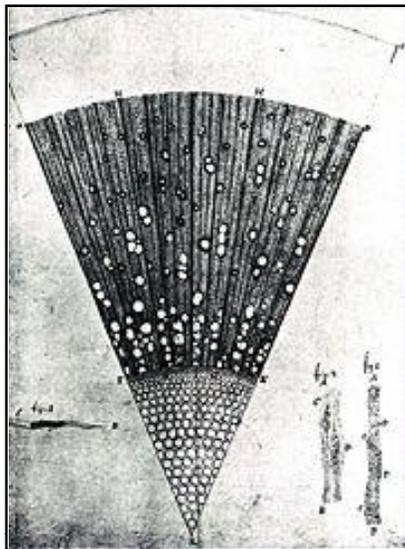
\* presented by Douglas Lanman, adapted from slides by Marc Levoy: <http://graphics.stanford.edu/talks/>

# Computational Imaging in the Sciences



## Driving Factors:

- new instruments lead to new discoveries (e.g., Leeuwenhoek + microscopy → microbiology)
- Q: most important instrument in last century?  
A: the digital computer



## What is Computational Imaging?

- according to B.K.P. Horn:  
*“...imaging methods in which computation is inherent in image formation.”*
- digital processing has led to a revolution in medical and scientific data collection (e.g., CT, MRI, PET, remote sensing, etc.)

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# Computational Imaging in the Sciences

## Medical Imaging:

- transmission tomography (CT)
- reflection tomography (ultrasound)

## Geophysics:

- borehole tomography
- seismic reflection surveying

## Applied Physics:

- diffuse optical tomography
- diffraction tomography
- scattering and inverse scattering

## Biology:

- confocal microscopy
- deconvolution microscopy

## Astronomy:

- coded-aperture imaging
- interferometric imaging

## Remote Sensing:

- multi-perspective panoramas
- synthetic aperture radar

## Optics:

- wavefront coding
- light field photography
- holography

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# Medical Imaging

## Overview:

- (non-invasive) reconstruction of internal structures of living organisms
- generally involves solving an inverse problem
- includes: radiology, endoscopy, thermal imaging, microscopy, etc.

## Methods:

- tomography (includes: CT, MRI, PET, ultrasound)
- electroencephalography (EEG) and magnetoencephalography (MEG)

## Key Issues:

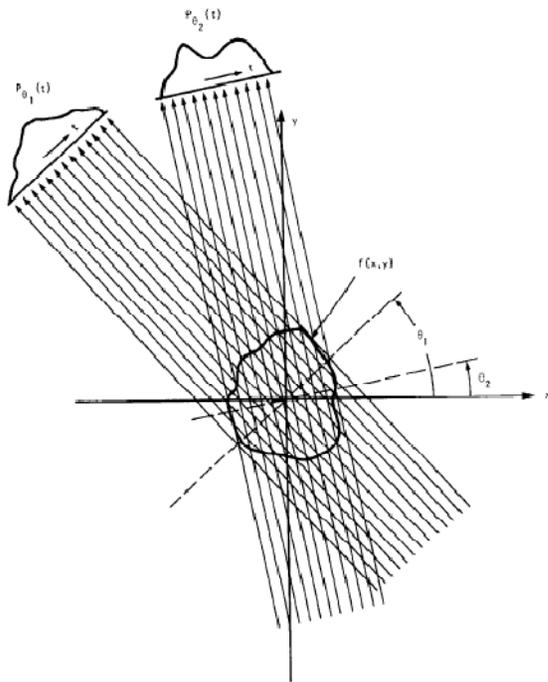
- safety (e.g., ionizing radiation)
- minimally invasive
- temporal and spatial resolution
- cost (to a lesser degree)

Two brain MRI images removed  
due to copyright restrictions.

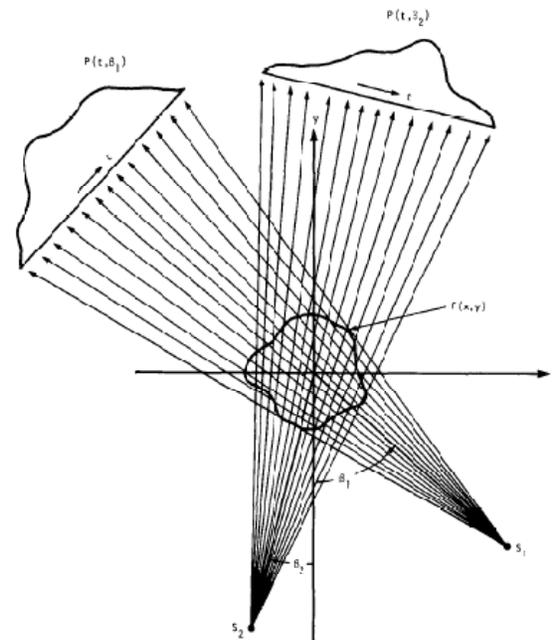
# What is Tomography?

## Definition:

- imaging by sectioning (from Greek *tomos*: “a section” or “cutting”)
- creates a cross-sectional image of an object by transmission or reflection data collected by illuminating from many directions



Parallel-beam Tomography



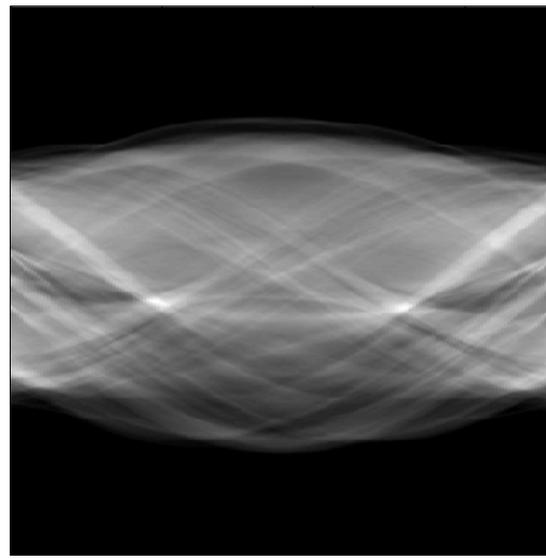
Fan-beam Tomography

\* Fig 3.2 and 3.3 in A.C. Kak and M. Slaney, *Principles of Computerized Tomographic Imaging*, 1988

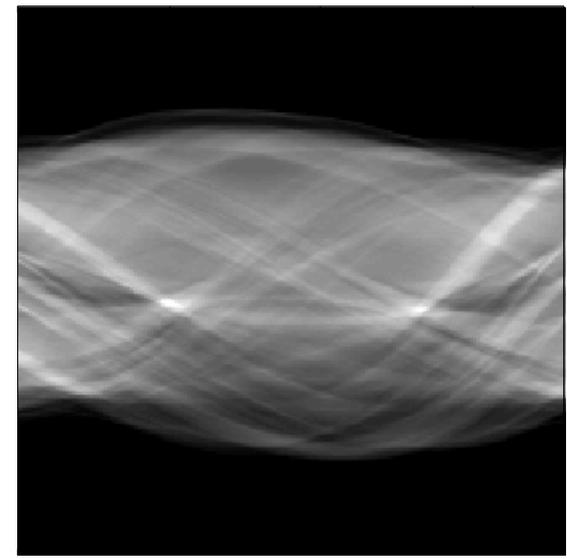
# Simulated Tomograms



density function

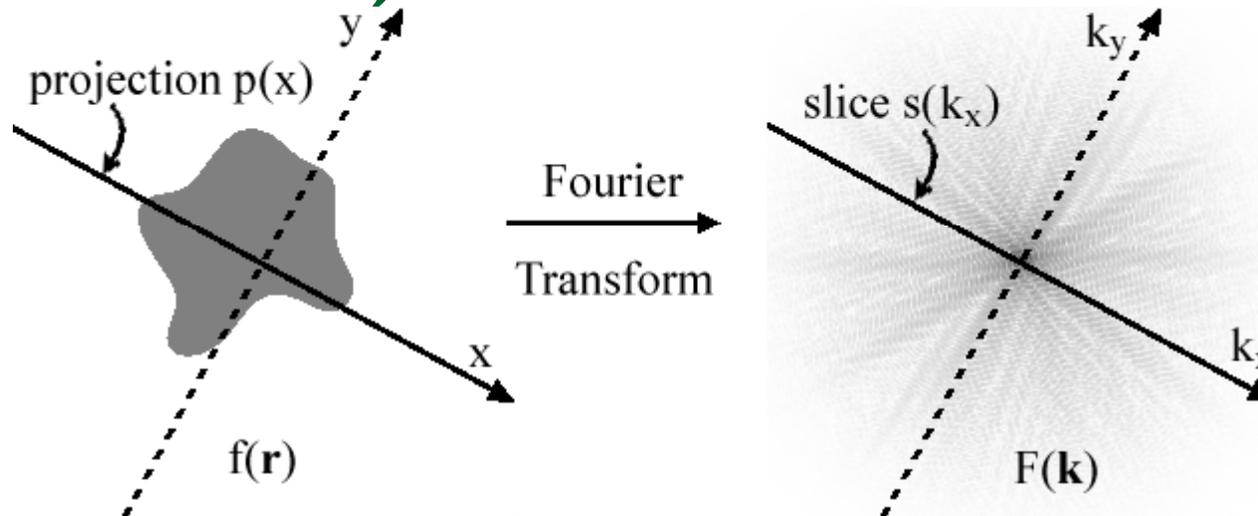


parallel-beam projections  
(Radon transform)



fan-beam projections

# Fourier Projection-Slice Theorem



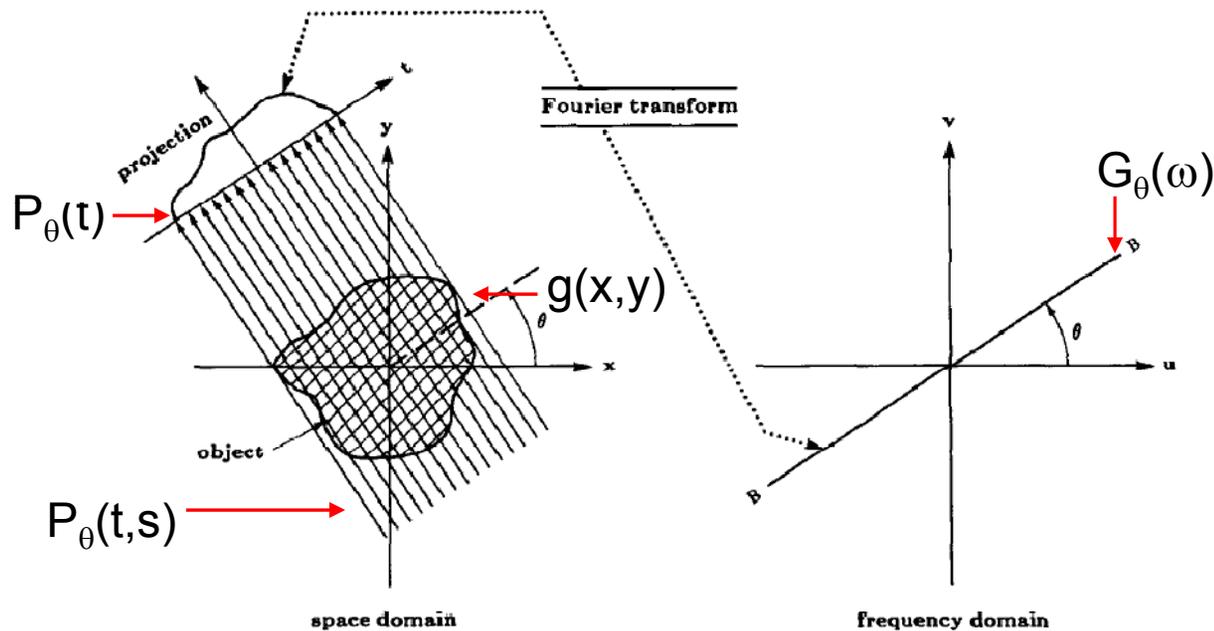
$$p(x) = \int_{-\infty}^{\infty} f(x, y) dy.$$

$$F(k_x, k_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{-2\pi i(xk_x + yk_y)} dx dy.$$

$$\begin{aligned} s(k_x) &= F(k_x, 0) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{-2\pi i x k_x} dx dy \\ &= \int_{-\infty}^{\infty} \left[ \int_{-\infty}^{\infty} f(x, y) dy \right] e^{-2\pi i x k_x} dx \\ &= \int_{-\infty}^{\infty} p(x) e^{-2\pi i x k_x} dx \end{aligned}$$

\* Image: Wikipedia, *Projection-Slice Theorem*, retrieved on 11/03/2008 (public domain);  
A.C. Kak and M. Slaney, *Principles of Computerized Tomographic Imaging*, 1988

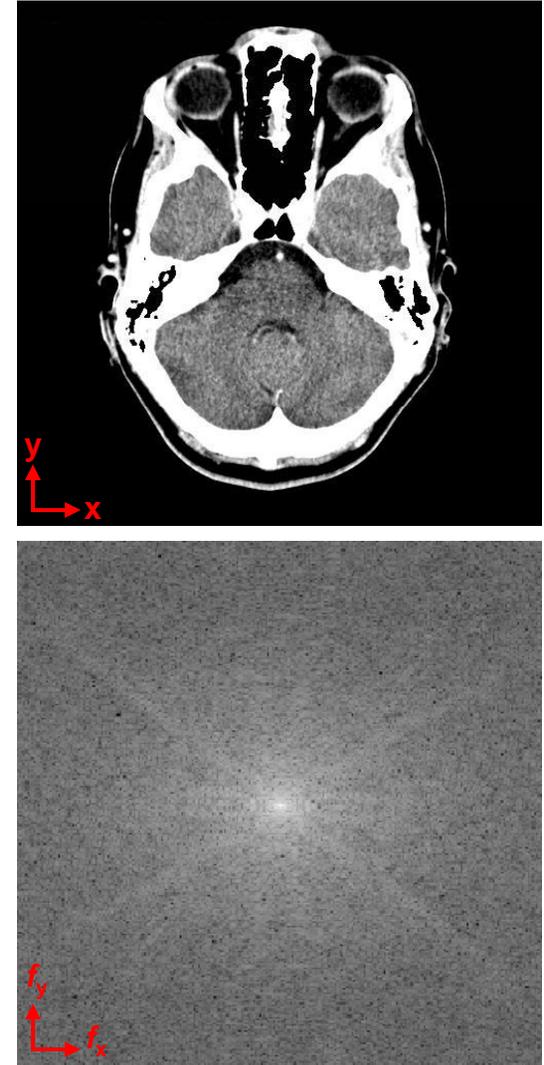
# Reconstruction: Filtered Backprojection



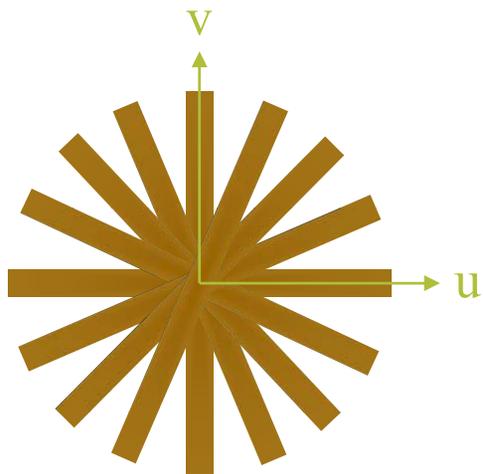
Courtesy of A. C. Kak and Malcolm Slaney. Used with permission.

## Fourier Projection-Slice Theorem:

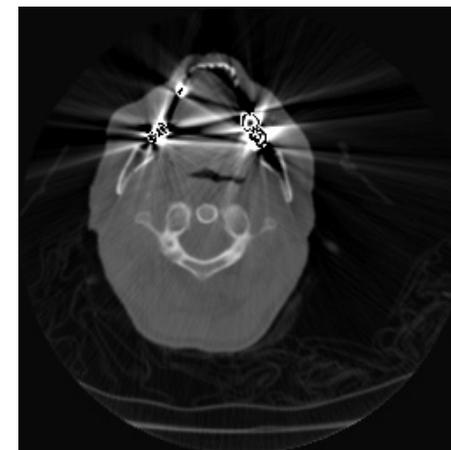
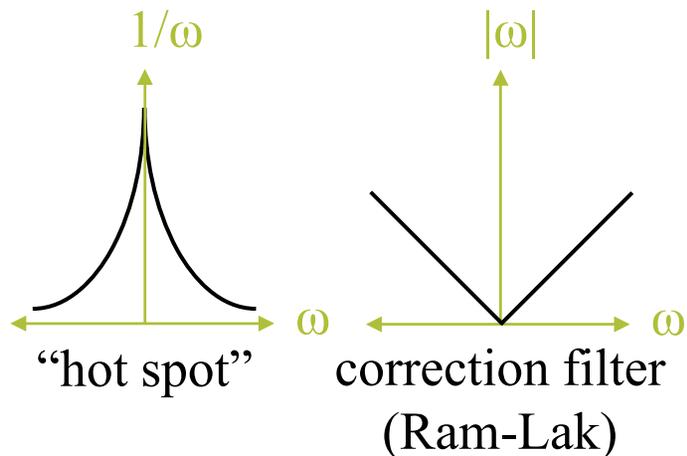
- $F^{-1}\{G_{\theta}(\omega)\} = P_{\theta}(t)$
- add slices  $G_{\theta}(\omega)$  into  $\{u,v\}$  at all angles  $\theta$  and inverse transform to yield  $g(x,y)$
- add 2D backprojections  $P_{\theta}(t)$  into  $\{x,y\}$  at all angles  $\theta$



# Sampling Requirements and Limitations



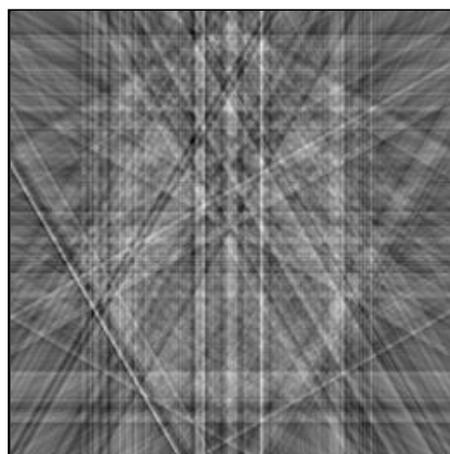
Diagrams courtesy of Marc Levoy.



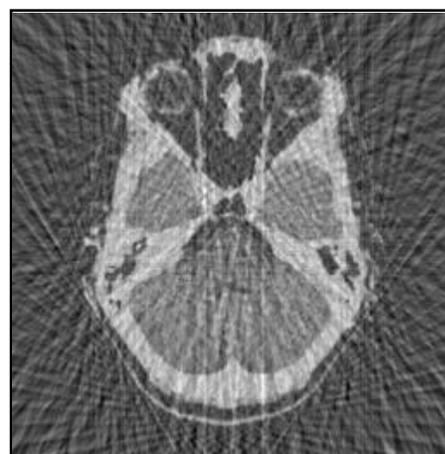
effect of occlusions



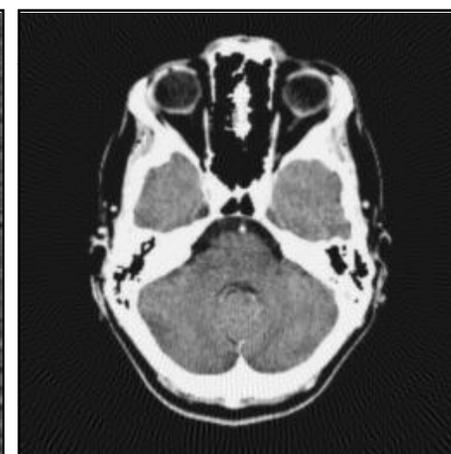
original density



30 deg. spacing



5 deg. spacing



1 deg. spacing

# Medical Applications of Tomography

Video still removed due to copyright restrictions.  
See Tuttle995i. "CT at max speed." March 1, 2008.  
<http://www.youtube.com/watch?v=2CWpZKuy-NE>

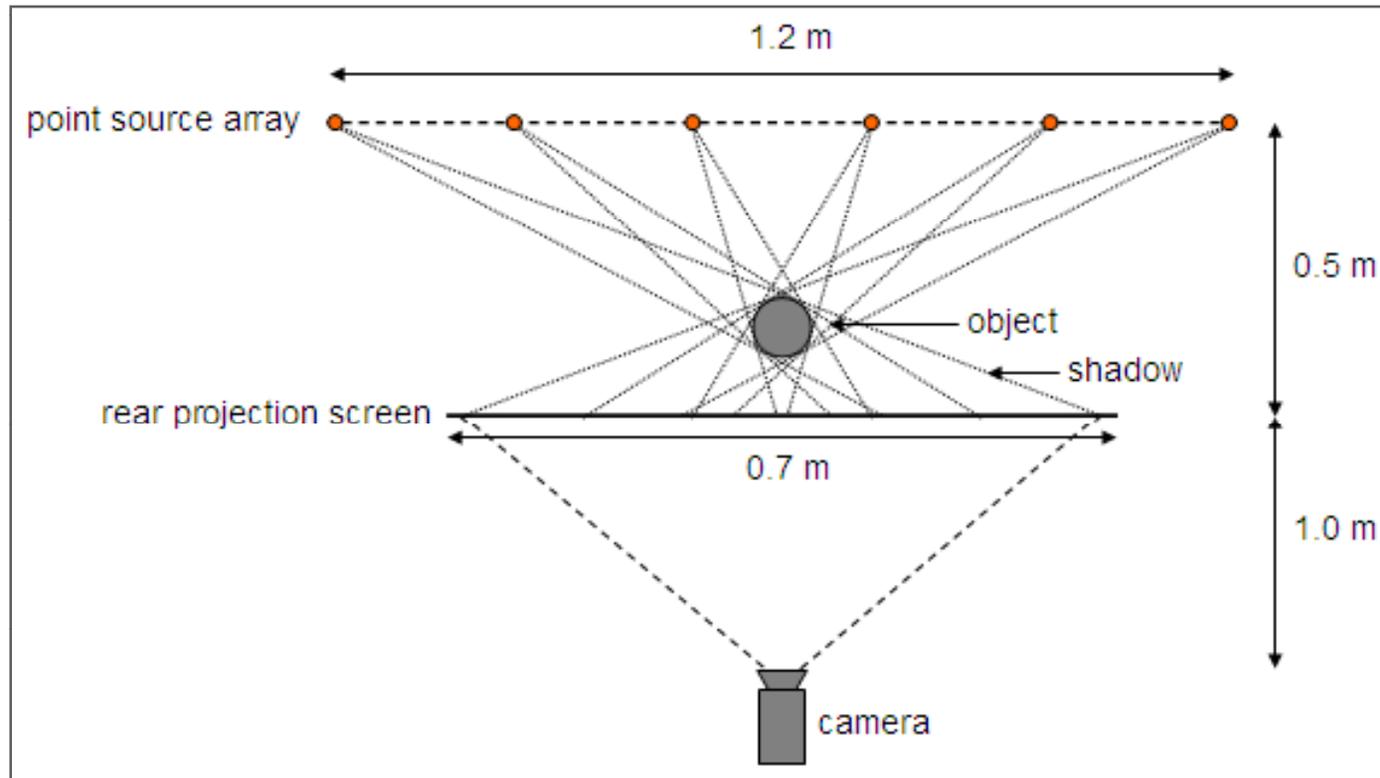
Medical images removed due to  
copyright restrictions.

Physical phenomenon	Type of tomograph
X-rays	CT
gamma rays	SPECT
electron-positron annihilation	PET
nuclear magnetic resonance	MRI
ultrasound	ultrasonography
electrons	3D TEM
ions	atom probe

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bone reconstruction    segmented vessels

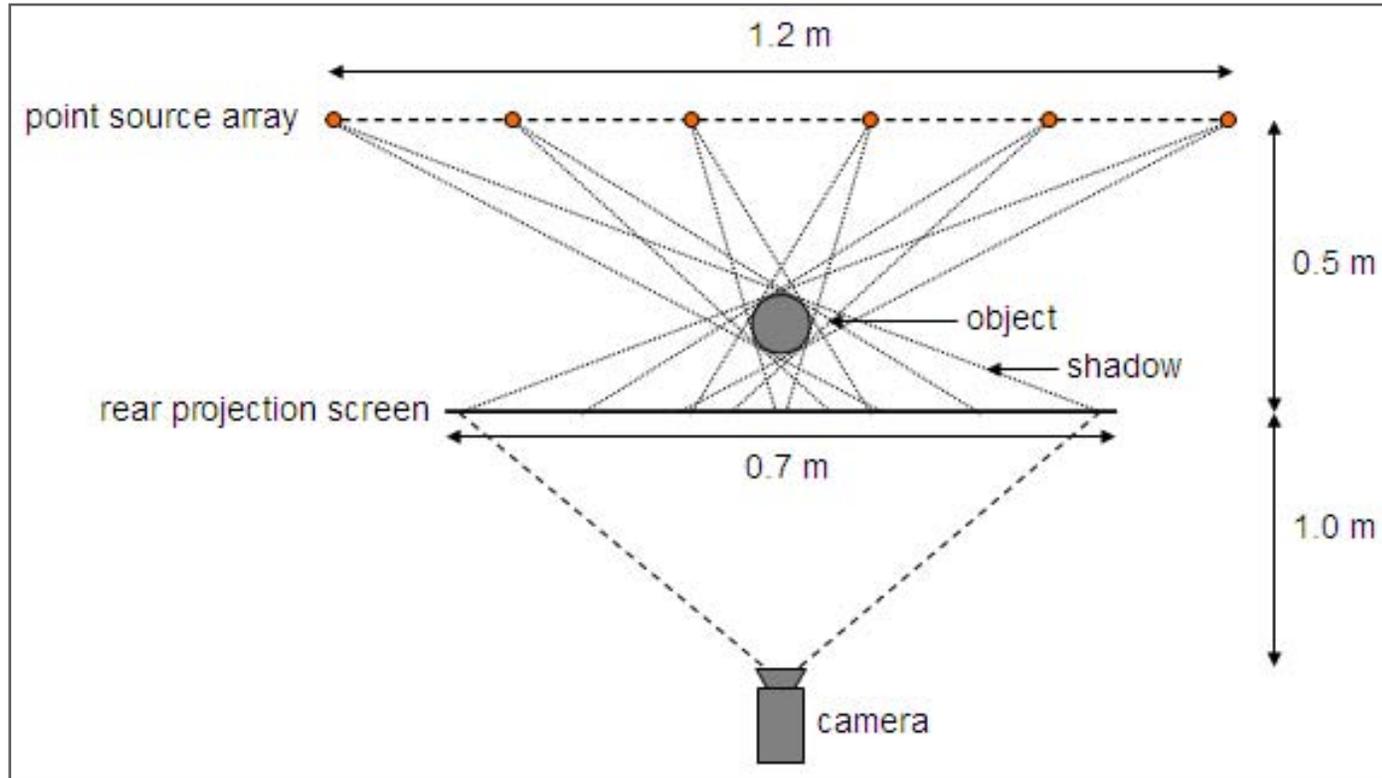
# How to Eliminate Moving Parts?



## Prior CT Generations: Time-sequential Projections

- moving parts
- synchronization and exposure timing
- costs (up-front, operating, and maintenance)
- dynamic scenes (RF interference, switching costs for X-ray tubes)

# How to Eliminate Moving Parts?

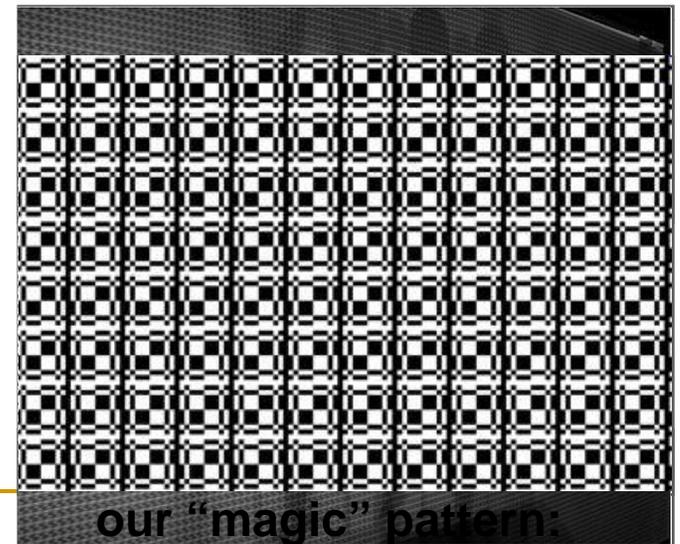
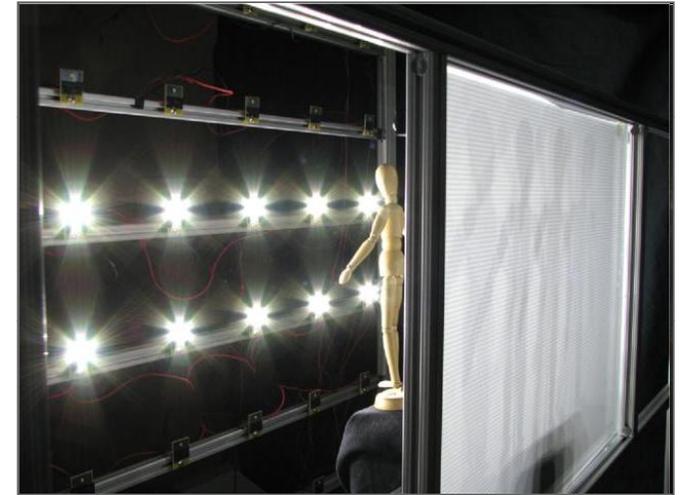


## Our Solution: Simultaneous Projections

- no moving parts
- no synchronization → uses “always-on” sources
- lower costs (simpler mechanical construction and X-ray sources)
- well-suited for high-speed capture of dynamic scenes

# Shield Fields:

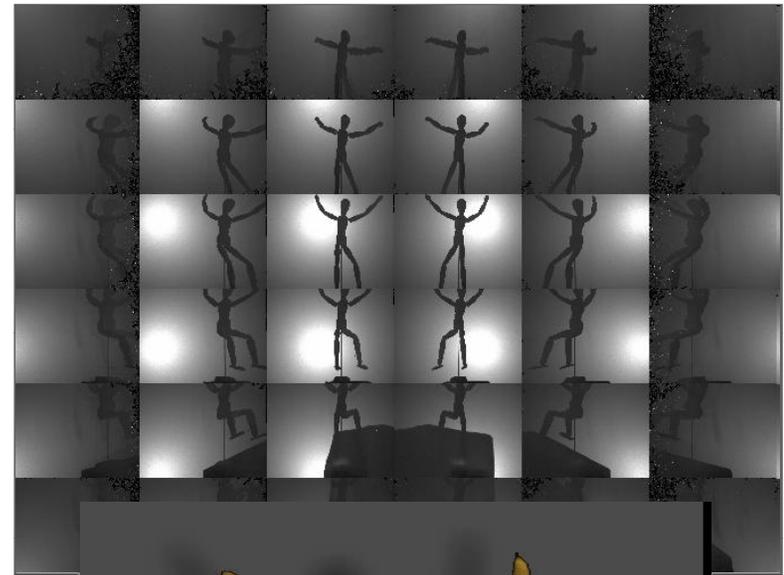
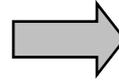
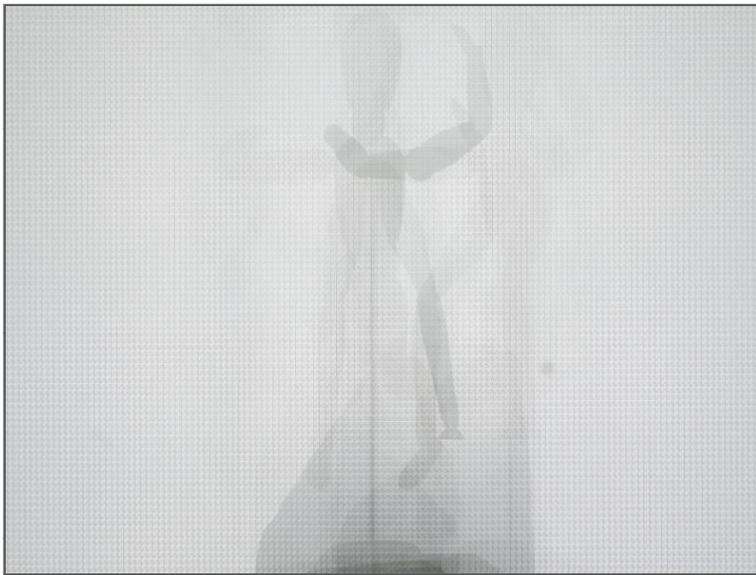
## Simultaneous Projections using Attenuating Masks



our "magic" pattern:  
allows spatial heterodyning

# Shield Fields:

## Simultaneous Projections using Attenuating Masks



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# Geophysical Applications

Two diagrams removed due to copyright restrictions.  
Borehole tomography and map of seismosaurus.  
From Reynolds, J.M., *An Introduction to Applied and Environmental Geophysics*. Wiley, 1997.

mapping a *seismosaurus* in sandstone  
using microphones in 4 boreholes and  
explosions along radial lines

## **Borehole Tomography:**

- receivers measure end-to-end travel time
- reconstruct to find velocities in intervening cells
- must use limited-angle reconstruction methods (e.g., ART)

# Geophysical Applications

Diagram removed due to copyright restrictions.

Borehole tomography.

From Reynolds, J.M., *An Introduction to Applied and Environmental Geophysics*. Wiley, 1997.

Photo removed due to copyright restrictions.

Stone map fragment from Stanford Forma Urbis Romae Project.

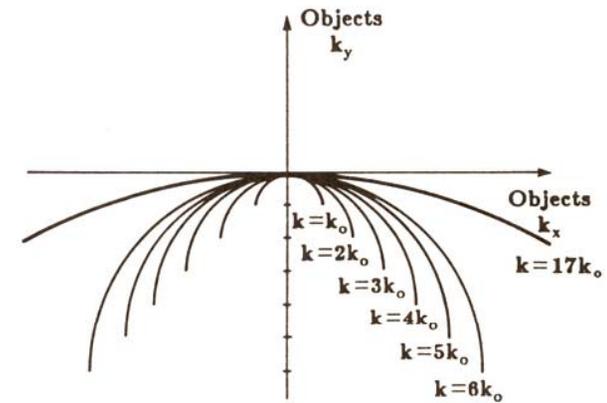
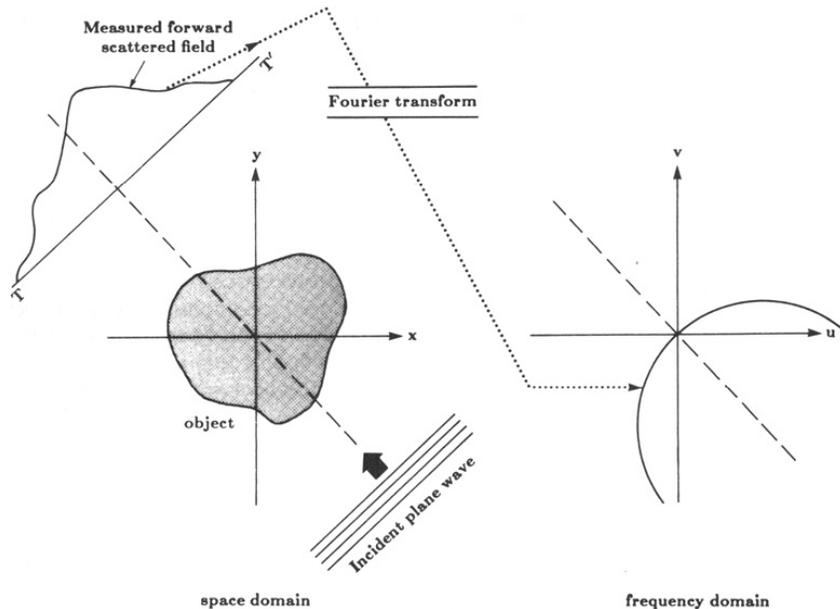
[http://formaurbis.stanford.edu/fragments/color\\_mos\\_reduced/010g\\_MOS.jpg](http://formaurbis.stanford.edu/fragments/color_mos_reduced/010g_MOS.jpg)

mapping ancient Rome using explosions in the subways and microphones along the streets?

## **Borehole Tomography:**

- receivers measure end-to-end travel time
- reconstruct to find velocities in intervening cells
- must use limited-angle reconstruction methods (e.g., ART)

# Optical Diffraction Tomography (ODT)



limit as  $\lambda \rightarrow 0$  (relative to object size) corresponds to Fourier Slice Theorem

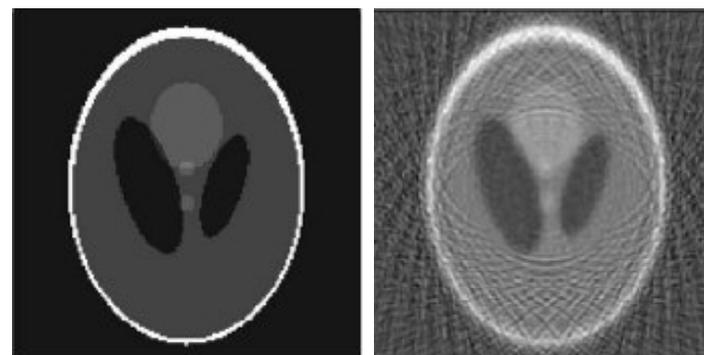
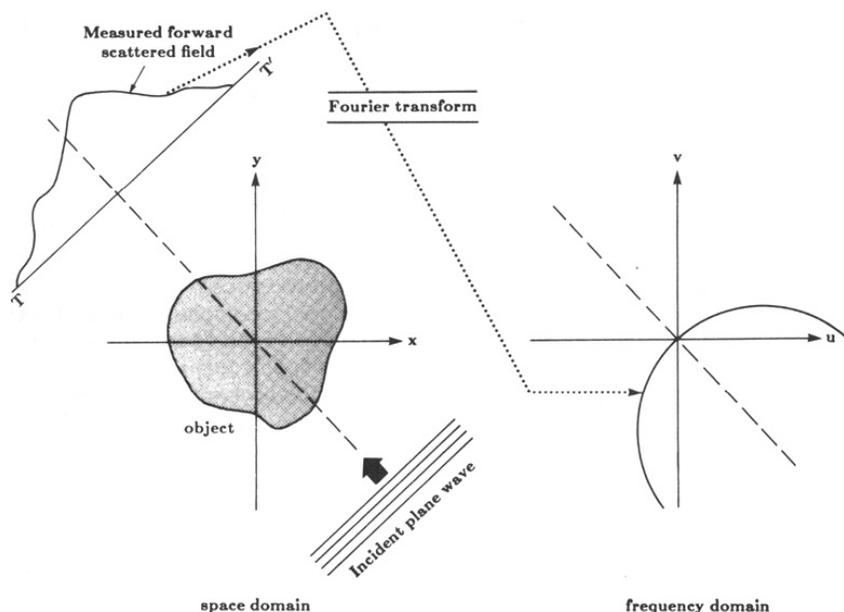
Courtesy of A. C. Kak and Malcolm Slaney. Used with permission.

- weakly-refractive media using coherent plane-wave illumination
- the amplitude and phase of the forward-scattered field can be described using the Fourier Diffraction Theorem, such that:  
 $\mathcal{F}\{\text{scattered field}\} = \text{arc in } \mathcal{F}\{\text{object}\}$
- repeat for multiple wavelength, use  $\mathcal{F}^{-1}$  to reconstruct volume
- broadband hologram will yield 3D structure (i.e., refraction indices)

Figures from A.C. Kak and M. Slaney, *Principles of Computerized Tomographic Imaging*, 1988

\* Slide derived from Marc Levoy

# Optical Diffraction Tomography (ODT)



synthetic ODT reconstruction example

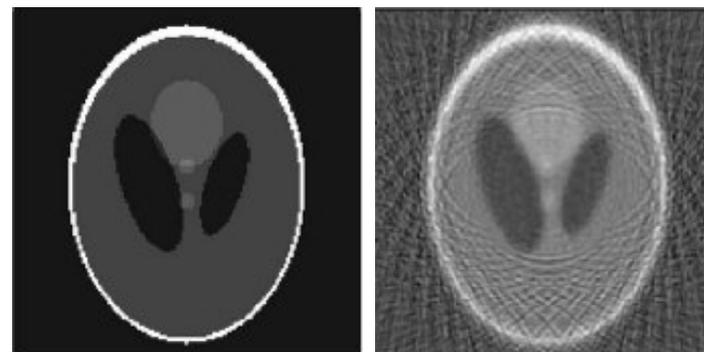
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Courtesy of A. C. Kak and Malcolm Slaney. Used with permission.

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# Optical Diffraction Tomography (ODT)

Diagram removed due to copyright restrictions. Source: Jebali, Asma. "Numerical Reconstruction of semi-transparent objects in Optical Diffraction Tomography." Diploma Project, Ecole Polytechnique, Lausanne, 2002.

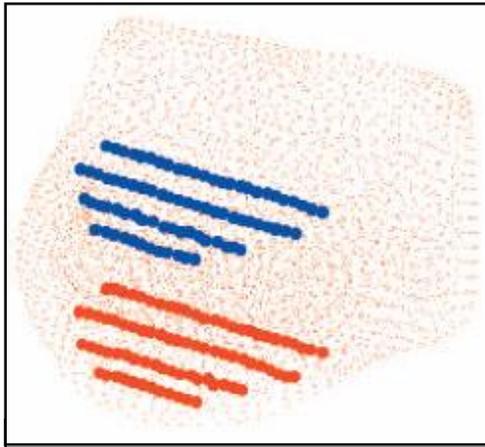


synthetic ODT reconstruction example

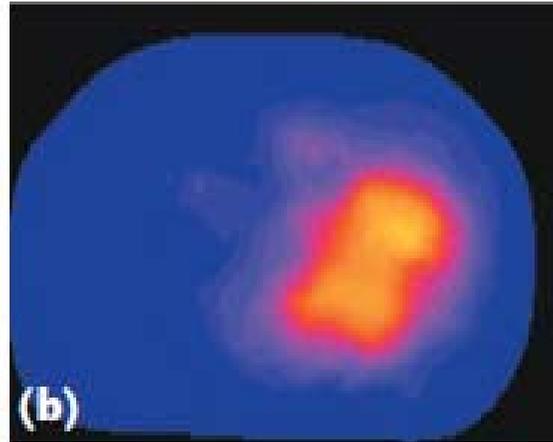
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- weakly-refractive media using coherent plane-wave illumination
- the amplitude and phase of the forward-scattered field can be described using the Fourier Diffraction Theorem, such that:  
 $\mathcal{F}\{\text{scattered field}\} = \text{arc in } \mathcal{F}\{\text{object}\}$
- repeat for multiple wavelength, use  $\mathcal{F}^{-1}$  to reconstruct volume
- backprojection possible using a depth-variant filter

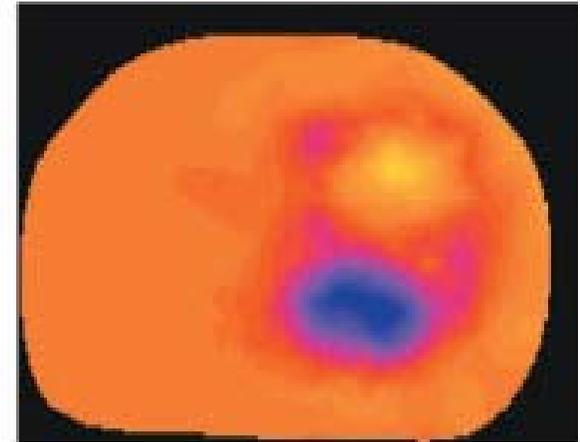
# Diffuse Optical Tomography (DOI/DOT)



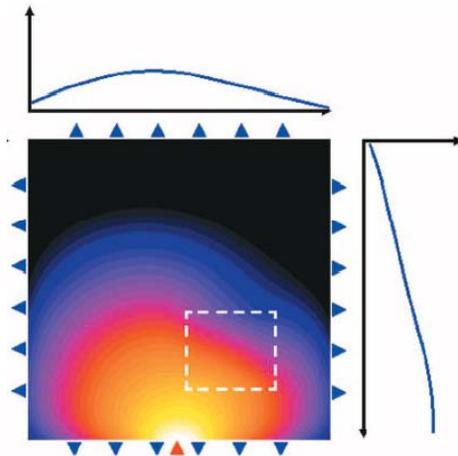
mammography application:  
sources (red), detectors (blue)



absorption (yellow is high)



Scattering (yellow is high)



- strongly-refractive media using inverse-scattering
- assumes propagation due to multiple scattering
- models transport as a diffusion process
- inversion is non-linear and ill-posed (i.e., hard)
- example: 81 emitters and 81 receivers, where time-of-flight gives initial estimate for absorption

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# Diffuse Optical Tomography (DOI/DOT)

Images removed due to copyright restrictions.

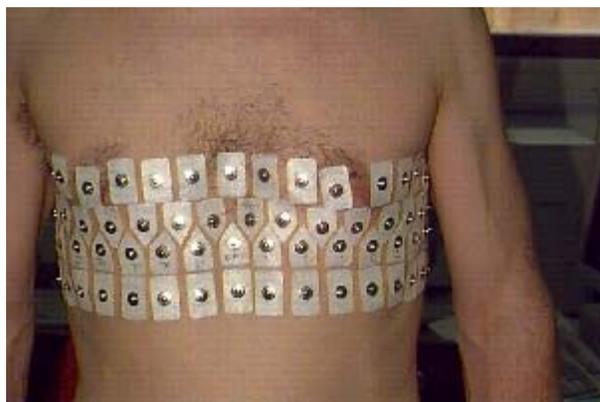
See Figure 4 in Gibson, A. P., et al. "Recent Advances in diffuse optical imaging." *Physics in Medicine and Biology* 50, no. 4 (2005): R1.

[http://www.medphys.ucl.ac.uk/research/borl/pdf/gibson\\_pmb\\_2005.pdf](http://www.medphys.ucl.ac.uk/research/borl/pdf/gibson_pmb_2005.pdf)

- strongly-refractive media using inverse-scattering
- assumes propagation due to multiple scattering
- models transport as a diffusion process
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# Electrical Impedance Tomography

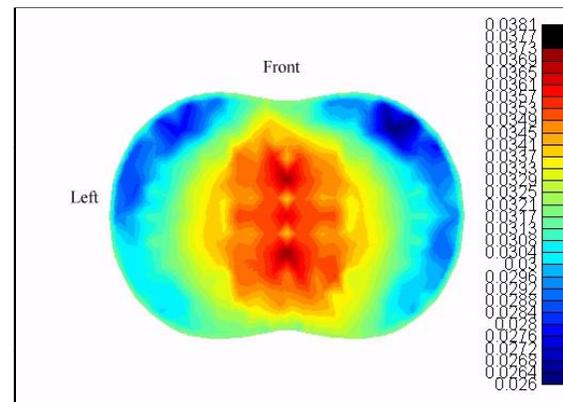
Kerrouche, N. et al. *Physiol. Meas.* 22 No 1 (February 2001) 147-157



chest electrode array

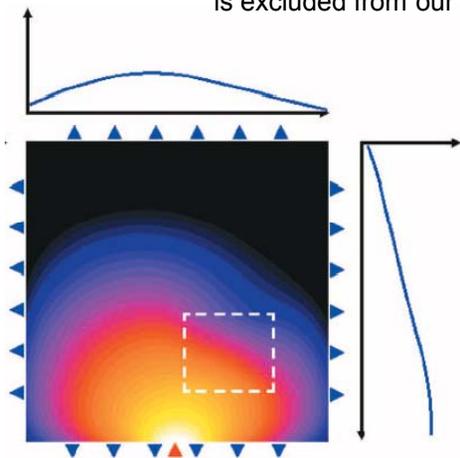


electrode array (wires attached)



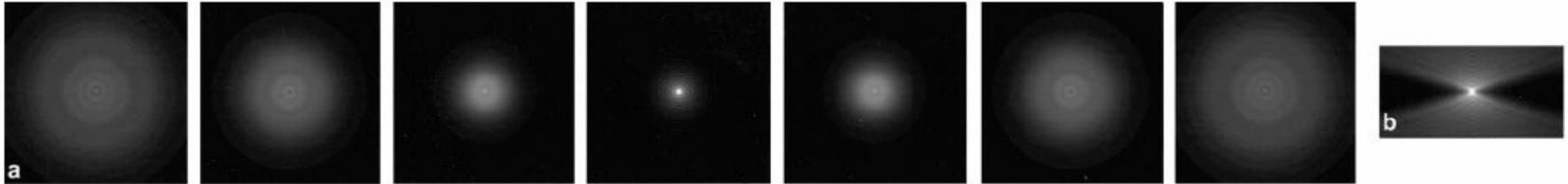
reconstructed conductivity map

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- strongly-refractive media using inverse-scattering
- assumes propagation due to multiple scattering
- models transport as a diffusion process
- inversion is non-linear and ill-posed (i.e., hard)
- EIT: inverse problem called “Calderón Problem”

# Biology: 3D Deconvolution



focal stack of a point in 3D is the 3D PSF of the imaging system

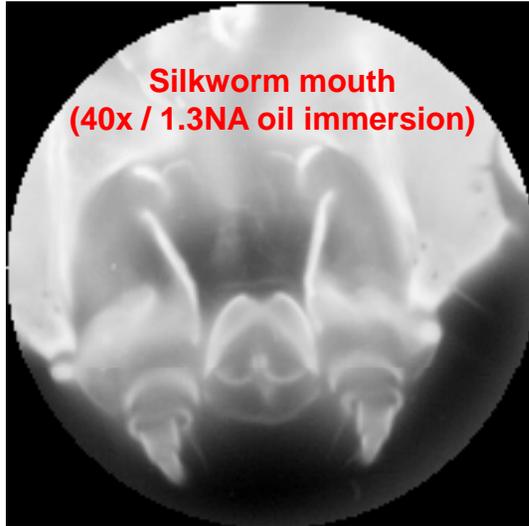
Courtesy of Elsevier, Inc., <http://www.sciencedirect.com>. Used with permission.

## Basics of 3D Deconvolution for Microscopy:

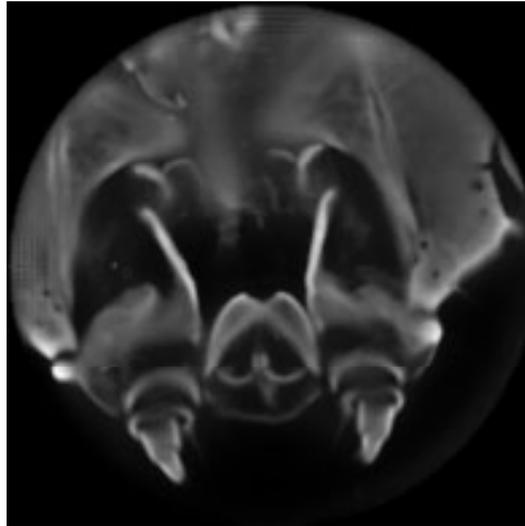
- object \* PSF  $\rightarrow$  focal stack
- $F\{\text{object}\} \times F\{\text{PSF}\} \rightarrow F\{\text{focal stack}\}$
- $F\{\text{focal stack}\} \div F\{\text{PSF}\} \rightarrow F\{\text{object}\}$
- spectrum contains zeros (due to missing rays)
- imaging noise is amplified by division by  $\sim$ zeros
- reduce by regularization (smoothing) or completion of spectrum
- improve convergence using constraints, e.g. object  $> 0$  (positivity)

# Biology: 3D Deconvolution

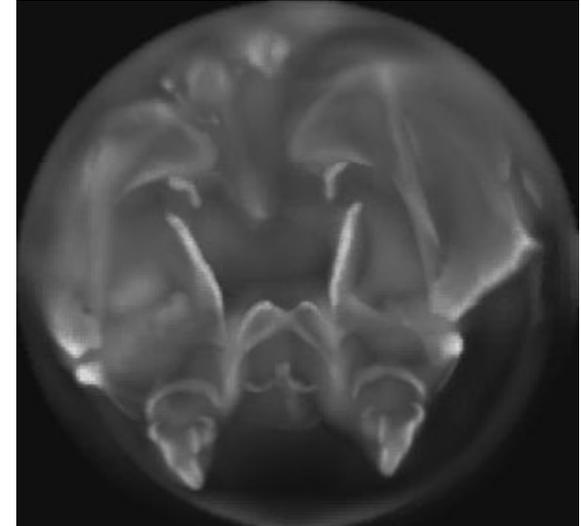
slice of focal stack



slice of volume

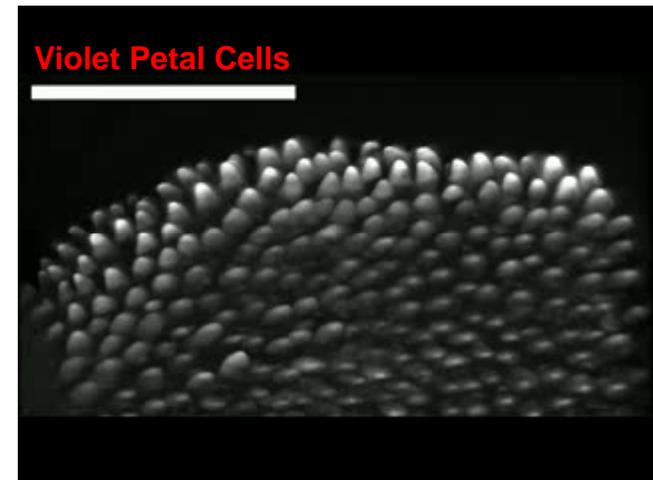


volume rendering

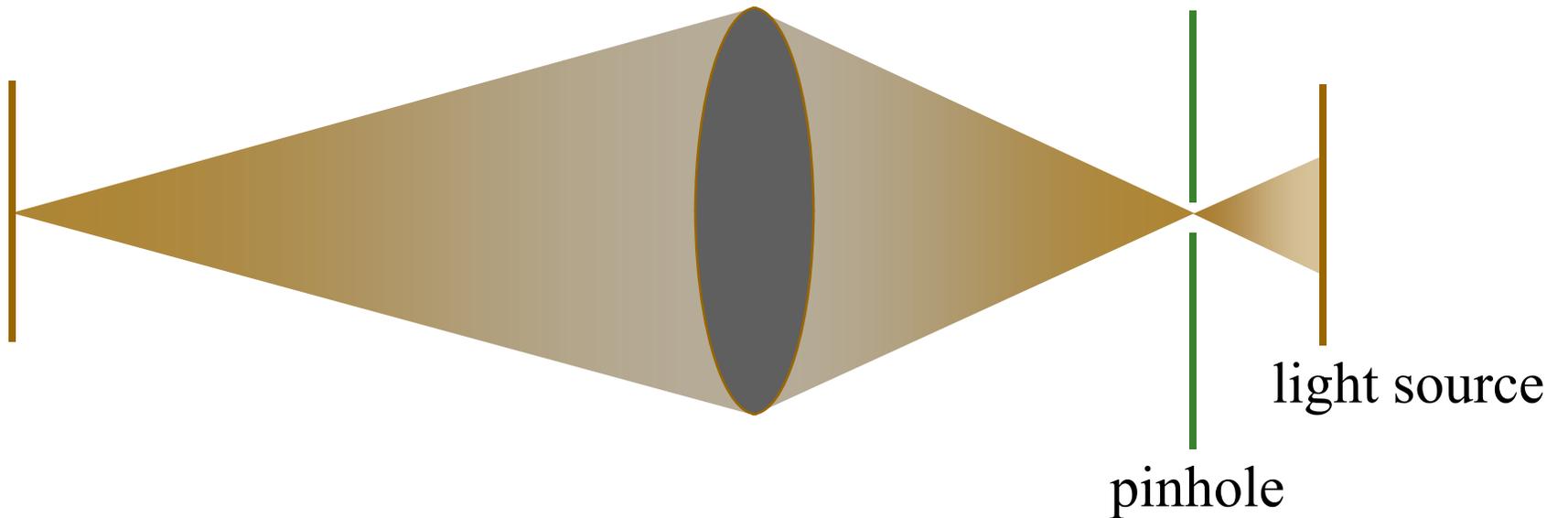


## 3D Deconvolution Microscopy:

- $\text{object} * \text{PSF} \rightarrow \text{focal stack}$
- $F\{\text{object}\} \times F\{\text{PSF}\} \rightarrow F\{\text{focal stack}\}$
- $F\{\text{focal stack}\} \div F\{\text{PSF}\} \rightarrow F\{\text{object}\}$



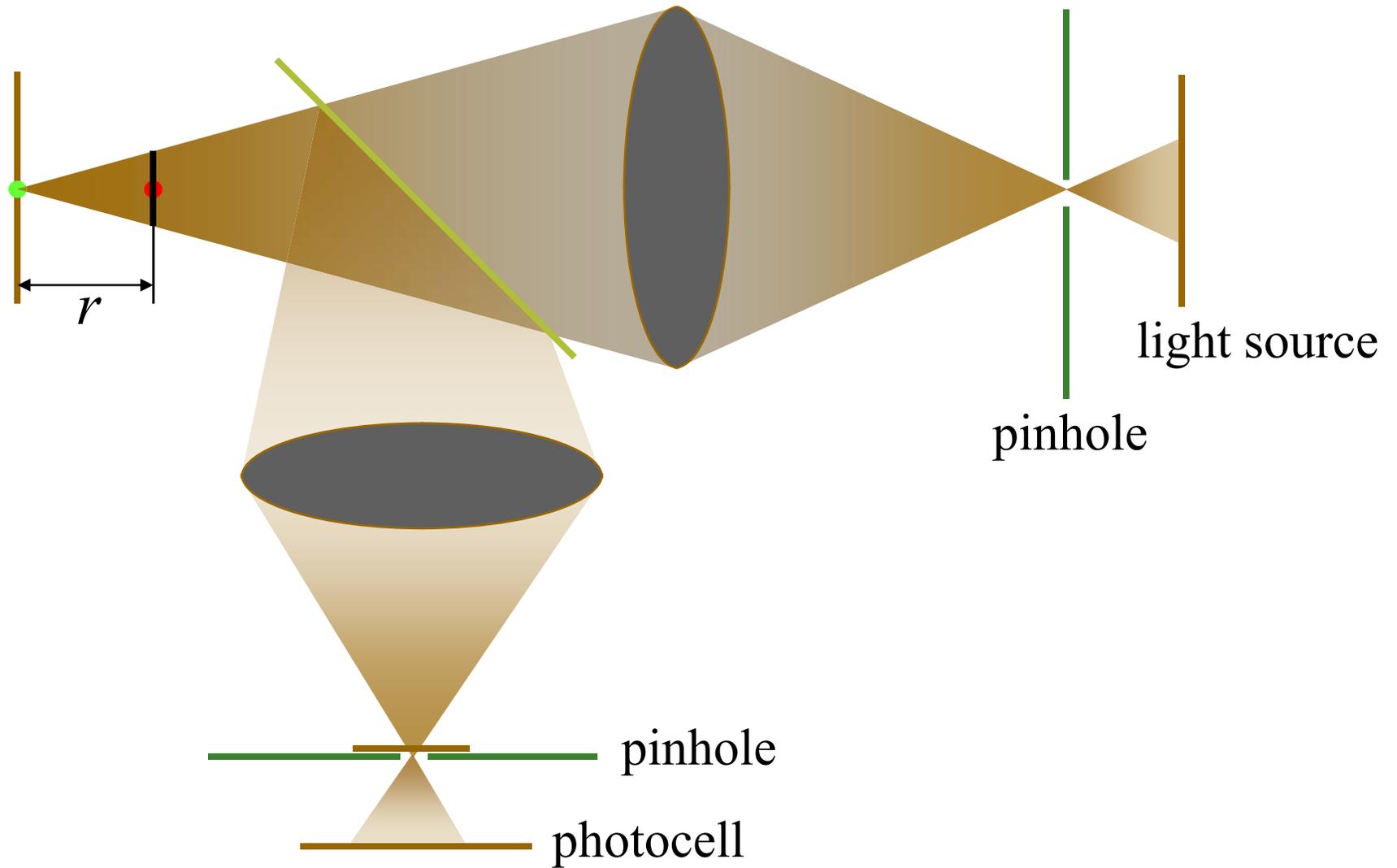
# Biology: Confocal Microscopy\*



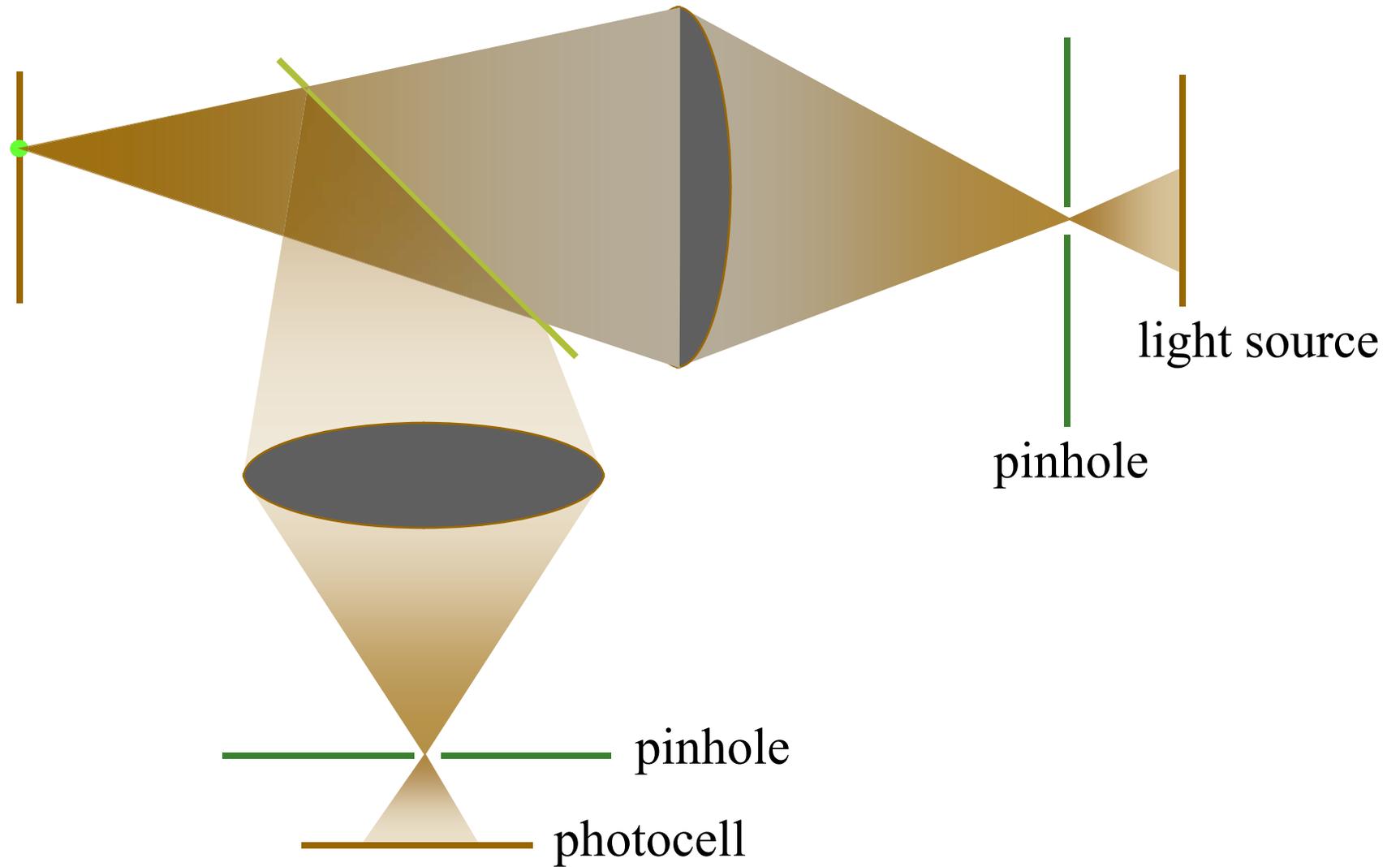
\* Slide derived from Marc Levoy

\* first patented by Marvin Minsky in 1957

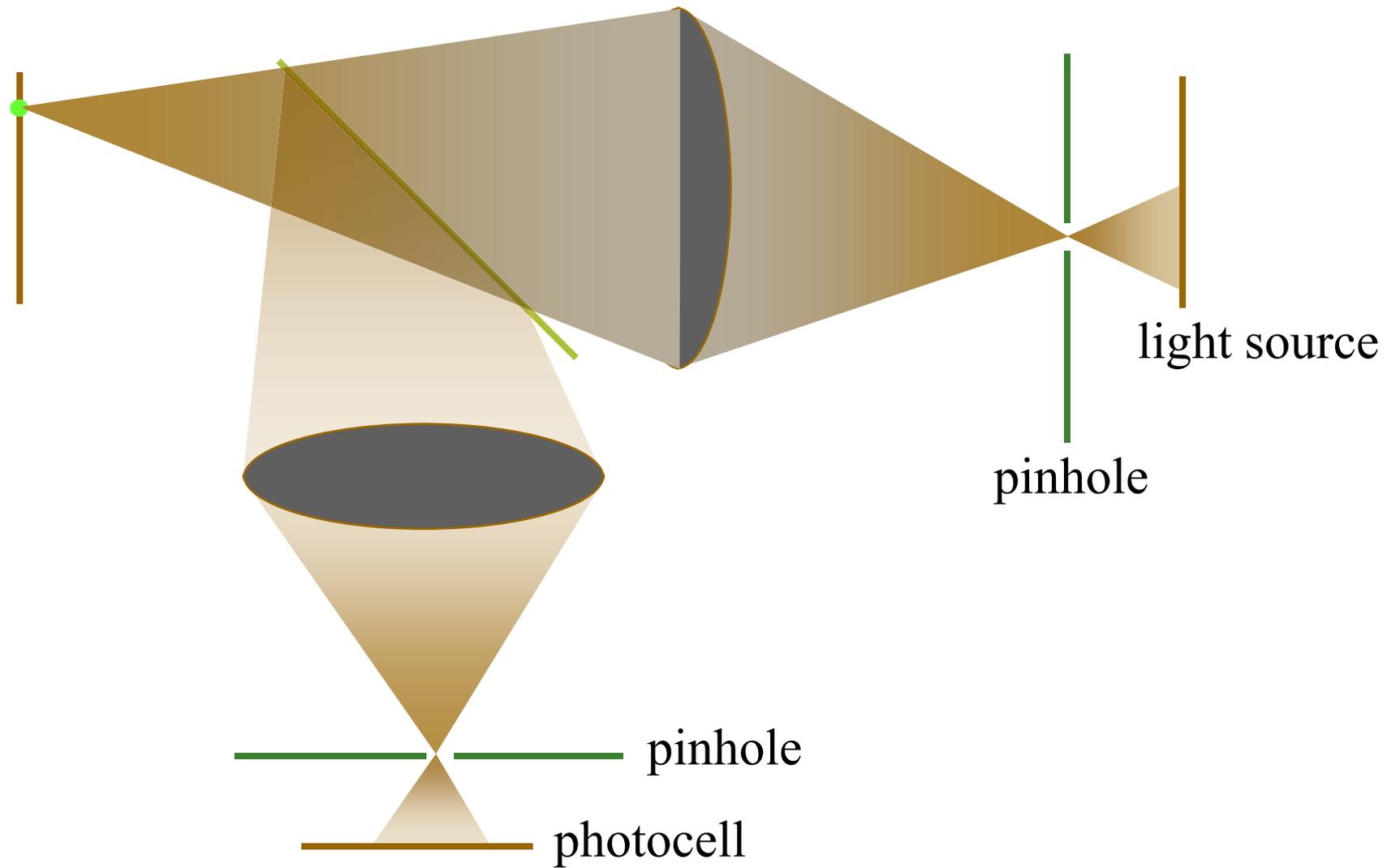
# Biology: Confocal Microscopy



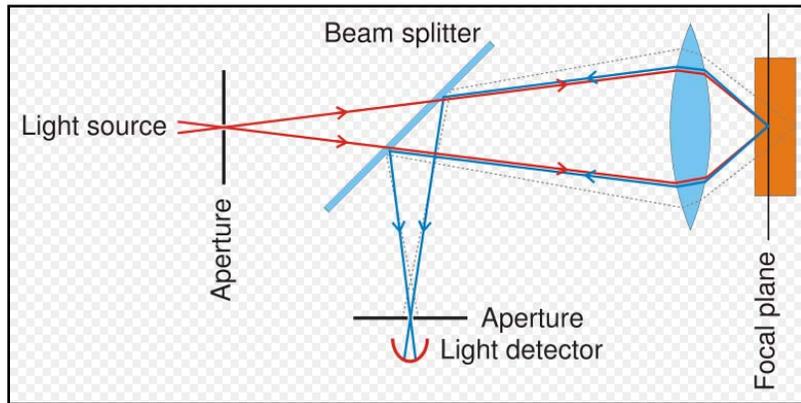
# Biology: Confocal Microscopy



# Biology: Confocal Microscopy



# Confocal Microscopy Examples



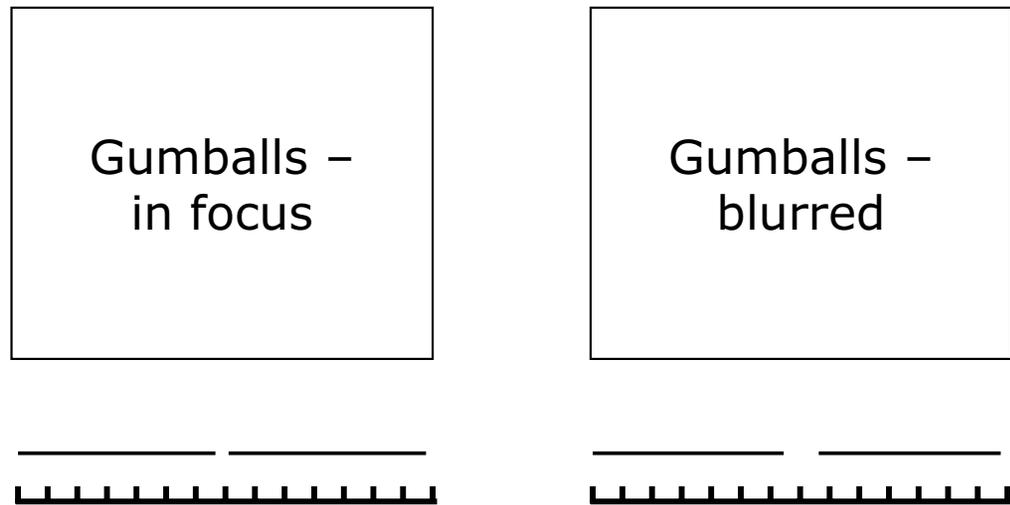
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Several confocal microscopy images removed due to copyright restrictions.

# Astronomy: Coded Aperture Imaging

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Diagrams removed due to copyright restrictions. Process for capturing and reconstructing a coded aperture image; schematic of a shielded detector.



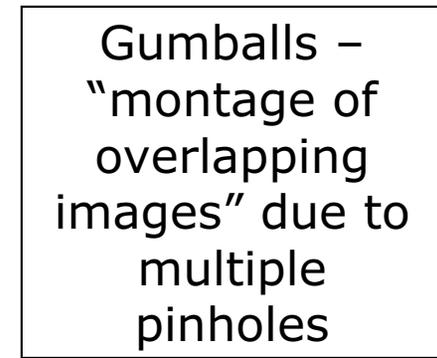
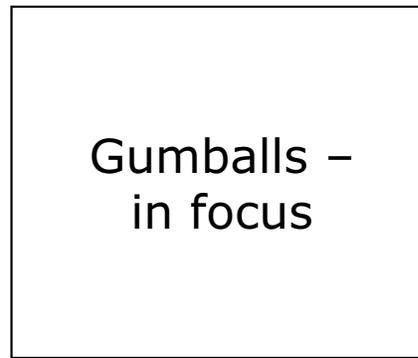
## Coded Aperture Imaging:

- cannot practically focus X-rays using optics
- pinhole would work, but requires long exposures
- instead, use multiple pinholes and a single sensor
- produces superimposed shifted copies of source

# Astronomy: Coded Aperture Imaging

Images removed due to copyright restrictions. See <http://www.paulcarlisle.net/codedaperture>

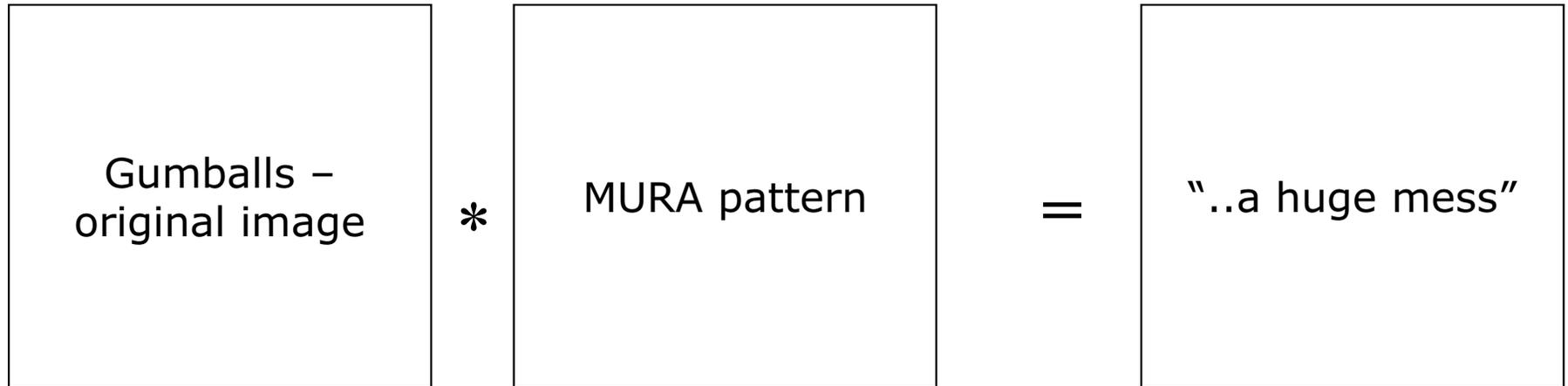
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## Coded Aperture Imaging:

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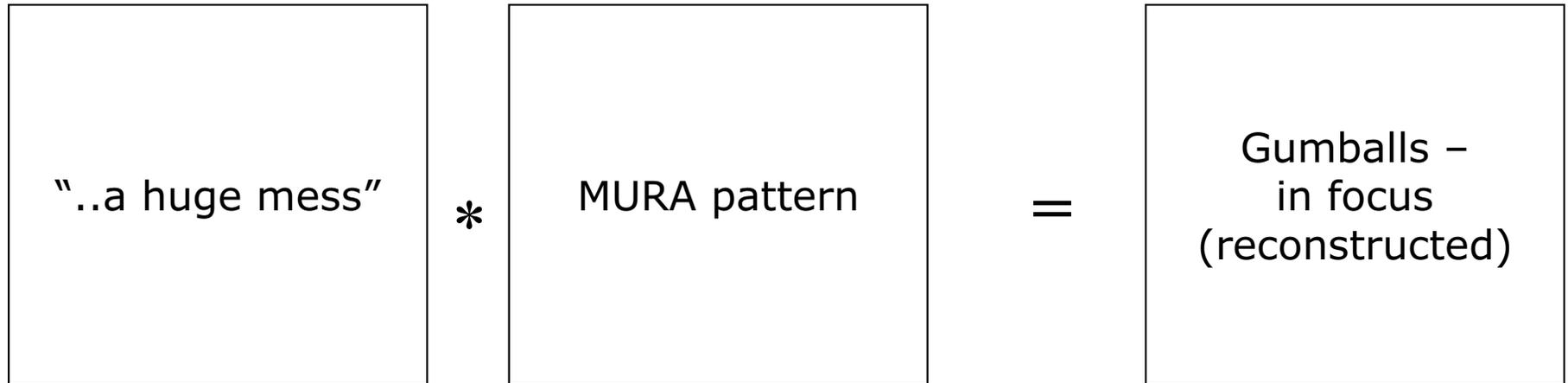


Images removed due to copyright restrictions. See <http://www.paulcarlisle.net/codedaperture>

## Coded Aperture Imaging:

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- produces superimposed shifted copies of source
- MURA: Modified Uniformly Redundant Array  
(key property: 50% open, autocorrelation function is a delta function)

# Astronomy: Coded Aperture Imaging



Images removed due to copyright restrictions. See <http://www.paulcarlisle.net/codedaperture>

## Coded Aperture Imaging:

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- MURA: Modified Uniformly Redundant Array  
(key property: 50% open, autocorrelation function is a delta function)

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# Near-Field Coded Aperture Imaging

Diagrams removed due to copyright restrictions. Process for capturing and reconstructing a coded aperture image; schematic of a shielded detector.

Diagram removed due to copyright restrictions. Schematic of flux passing through mask into detector.

## **Coded Aperture Imaging (Source Reconstruction):**

- backproject each detected pixel intensity through each hole in mask
- superimposition of projections reconstructs source (plus a bias)
- essentially a cross-correlation of detected image with mask
- also works for non-infinite sources (in which case, must use voxel grid)
- assumes sources are not occluded (except by the mask)

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MAS.531 / MAS.131 Computational Camera and Photography  
Fall 2009

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