Lecture # 18, Quantum Computation 2: Quantum Protocols And Communications

Lecture notes of Isaac Chuang, transcribed by Jennifer Novosad

Administrata:

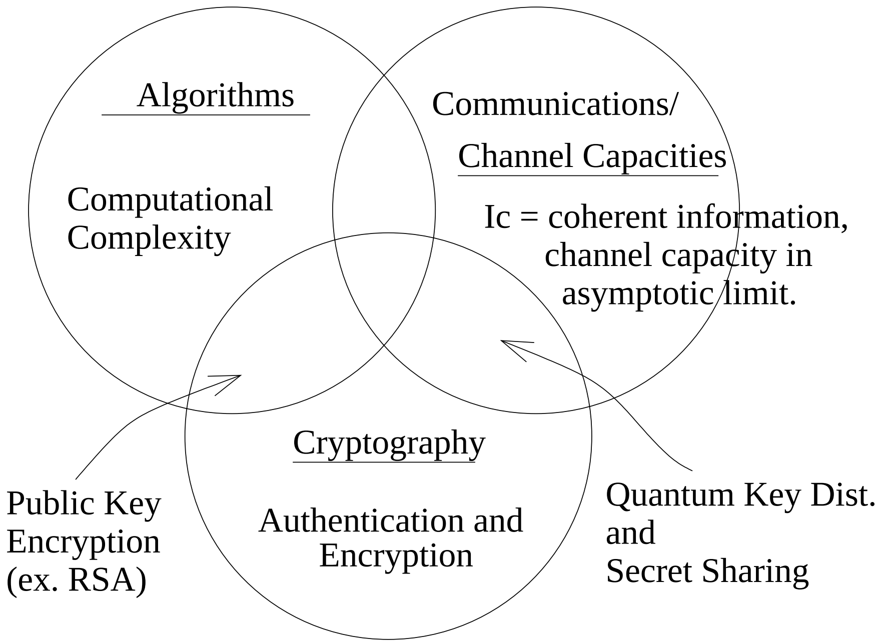
Projects will be presented in class in May, 3 presentations a day, 20 min slots.Sign up list is on the Wikki, first come first serve. Papers due May 12 (1/2 way between presentations).

Outline:

Tasks over a distributed set of parties

1. Perspective
2. Classical Communication Complexity
3. Ex. Fingerprinting (Q)
4. Digital Signatures
5. Q.D.S. Scheme

1. PERSPECTIVE



$I_c \simeq$ measure of trust

If a party shares a pure, entangled state, then it is only known to that party. So, it acts as a measure of trust.

How do you use it?

2. CLASSICAL COMMUNICATION COMPLEXITY

⇒ General setting

f is publicly known

$f : \{0,1\}^n \times \{0,1\}^n \mapsto \{0,1\}$

Alice: $x \in \{0,1\}^n$

Bob: $y \in \{0,1\}^n$

How much communication do they need to compute f?

⇒ Options:

(1) Classical or Quantum bits

(2) Can compute answer with

- exactly (no error)
- bounded error $\pm\epsilon$
- one sided error

  ...

(3) Assumptions

- Prior Shared Randomness (secret keys)
- Shared Entanglement

2.1. Example: Equality

$$f(x,y) = Eq(x,y) = \begin{matrix} 0 & \text{if x=y} \\ 1 & \text{if otherwise} \end{matrix}$$

$D(Eq) = n$ is deterministic or exact answer

$R(Eq) = ?$ is Random or Bounded error

2.1.1. Randomized Protocol:

⇒ Setup:

A and B agree on $P > n/\epsilon$, where ϵ is the error and P is a prime.

Locally compute 2 polynomials over the field $F(P)$,
$A(z) = x_1 + x_2 z + x_3 z^2 + ... + x_n z^{n-1}$
$B(z) = y_1 + y_2 z + y_3 z^2 + ... + y_n z^{n-1}$

Note for $C(z) = A(z) - B(z)$,
$x = y \to C(z) = 0$
$x \neq y \to$(# z's s.t. C(z)=0)$\leq n$

⇒ Protocol:

A chooses random $z \in F(P)$. Sends $(z, A(z))$

B computes $C(z)$, outputs EQ if $C = o$, NEQ otherwise.

⇒ Analysis

$Prob(C(z) = 0, x \neq y) \leq n/p < \epsilon$, by def. of P earlier.

A sends $2 \log P$ bits $= o(\log n - \log 1/\epsilon)$

R(EQ) $\sim O(\log n - \log 1/\epsilon) \cong O(\log n)$

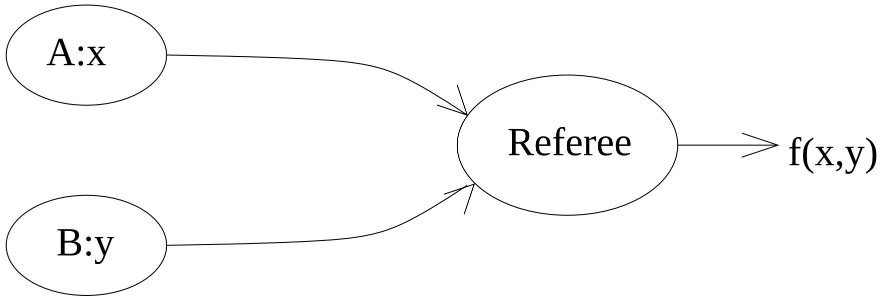
Approach: communicate the result of a function rather than the number itself.

2.2. Table of Costs

Problem	Exact Classical	R	Quantum Random	Quantum Exact
EQ	n	$\log n$	$\log n$	n
Parity/Inner Product	n	n	n	n
Dist J	n	n	$\sqrt{n}$	?
Distributed Deutsch Joza	n	$\log n$	$\log n$	$\log n$
RAZ (a promise function)		$n^{1/4}/\log n$	$\log n$	

3. FINGER PRINTING AS A SPECIFIC EXAMPLE

⇒ 3 party model, 1979, "simultaneous message passing" Andrew Yan



specific case where $f(x, y) = EQ(x, y)$

⇒ Classical: $\exists n \to m$ bit code with the following properties:

$\{E(x) \in \{0,1\}^m |\ x \in \{0,1\}^n,$

$m = cn,$

$dist(E(x), E(y)) \geq (1 - \delta)m$ if $x \neq y\}$

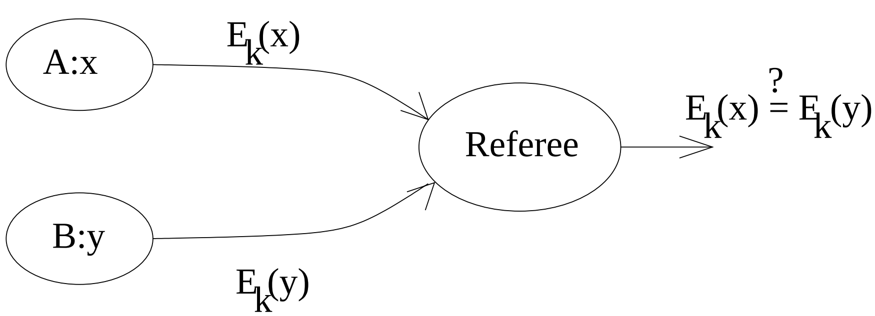
δ, c constants, $E(x)$ code words, $dist = \#$ different bits.

Example: Justen codes 1972

any $c > 2$, $\delta < \frac{9}{10} + frac115c$

Let $E_i(x)$ denote the i^{th} bit of the code word. Suppose Alice and Bob share a secret key $k \in \{0,1\}^{\log m}$

3.1. Protocol



Not Error:

$Prob_{correct}(E_k(x) \neq E_k(y) | x \neq y) \geq 1 - \delta$

Boosting: Repeat the protocol r times, with r values of k. $Prob_{error} \to \delta^r$

Disadvantage: Need r secret keys

$\Rightarrow$ with no secret keys? Open problem from Yao, 1979

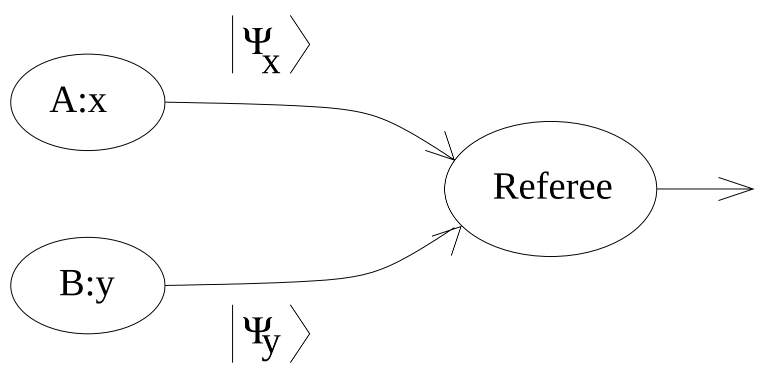
Solved in 1996, Ambians, Neuman to Szegedy, Babai

Requires $\Omega(\sqrt{n})$ bits

$\Rightarrow$ Quantum Protocol for same problem

needs $O(\log n)$ qubits without secret key

Buhrman, Cleve, Watrow 2001



where $|\Psi_x\rangle$ and $|\Psi_y\rangle$ are quantum bit strings.

3.2. Two Theorems

3.2.1. Thm 1

$\exists\ 2^{2^m}$ states $|\Psi_x\rangle$ of m qubits such that $\langle\Psi_{x'}|\Psi_x\rangle \leq \delta$ for $x' \neq x$ and δ const (for some δ). (so, the states are not quite orthogonal.)

Proof:

Let $|\Psi_x\rangle = \frac{1}{\sqrt{m}} \sum_{k=o}^{m-1} |E_k(x)\rangle |k\rangle$

Then:

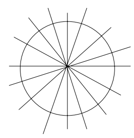
$\langle\Psi_x|\Psi_x\rangle = 1$

$x \neq y \rightarrow \langle \Psi_x | \Psi_y \rangle = \frac{1}{m} \sum_{kk'} \langle E_k(x) | E'_k(y) \rangle \langle k | k' \rangle$

$= \frac{1}{m} \sum_{k} \langle E_k(x) | E_k(y) \rangle$

$\leq m\delta / m = \delta$ bits the same, though x and y differ

Note: stabilizers also work, or states on the unit circle



3.2.2. Thm 2

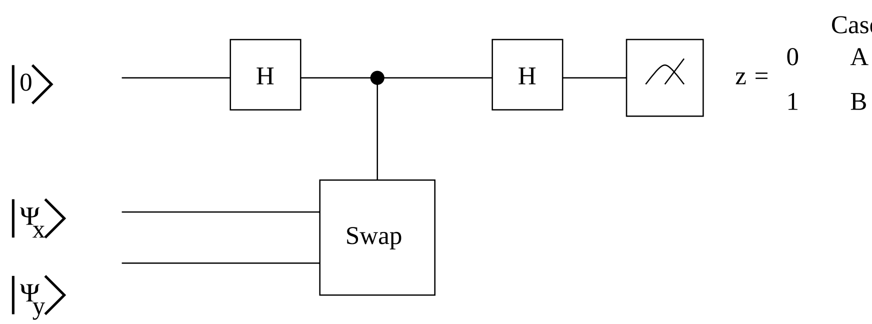
no-cloning $\Rightarrow$ no perfect equality test.

Given 2 states $|\Psi_x\rangle$ and $|\Psi_y\rangle$ s.t. either $|\Psi_x\rangle = |\Psi_y\rangle$ or $|\Psi_x\rangle \neq |\Psi_y\rangle$ and $|\langle \Psi_x | \Psi_y \rangle| \leq \delta$,

Then which case is true can be determined with probability of error $\leq \frac{1+\delta^2}{2}$

Proof:

Try to measure the operator swap



$|0, \Psi_x, \Psi_y\rangle \rightarrow |0+1, \Psi_x, \Psi_y\rangle$

$\rightarrow |0, \Psi_x, \Psi_y\rangle + |1, \Psi_x, \Psi_y\rangle$

$\rightarrow |0+1, \Psi_x, \Psi_y\rangle + |0-1, \Psi_x, \Psi_y\rangle$

$= |0\rangle(|\Psi_x, \Psi_y\rangle + |\Psi_y, \Psi_x\rangle) + |1\rangle(|\Psi_x \Psi_y\rangle - |\Psi_y, \Psi_x\rangle)$

First portion is symmetric case, second portion is antisymmetric case, $= |\phi\rangle$

$Prob(z = 1 | x \neq y) = \frac{1}{4} |\langle \phi | \phi \rangle|^2$

$= \frac{1}{4} |(\langle \Psi_x \Psi_y | - \langle \Psi_y \Psi_x |)(|\Psi_x \Psi_y\rangle - |\Psi_y \Psi_x\rangle)|$

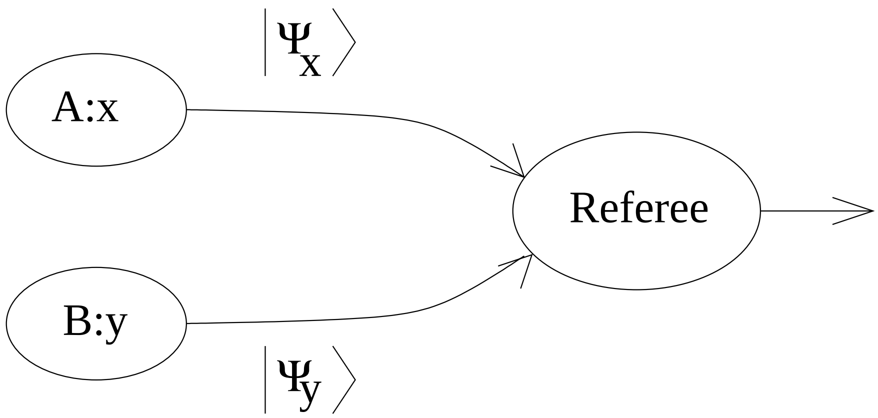
$= \frac{1}{4}(2 - 2|\langle\Psi_x|\Psi_y\rangle|^2)$

$\leq \frac{1}{4}(2 - 2\delta^2) = \frac{1}{2}(1 - \delta^2)$

Probability of Error $\leq \frac{1}{2}(1 - \delta^2)$

Probabilistic equality test that requires a promise.

Finger Printing Protocol:



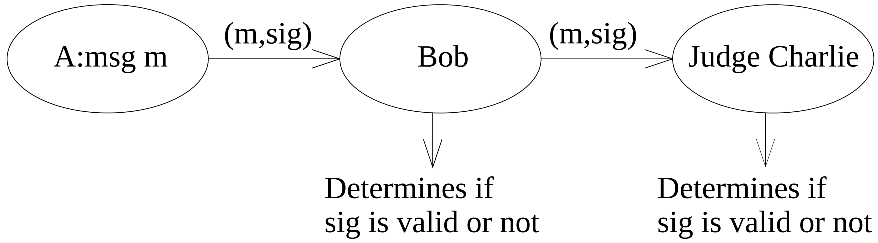
Probability of Error $\leq \frac{1}{2}(1 - \delta^2) =$ const.

Can boost this by repeating $O(\log \frac{1}{\epsilon})$ times $\rightarrow P_{error} < \epsilon$

Concept: Replaced shared randomness with qubits (this does not always work).

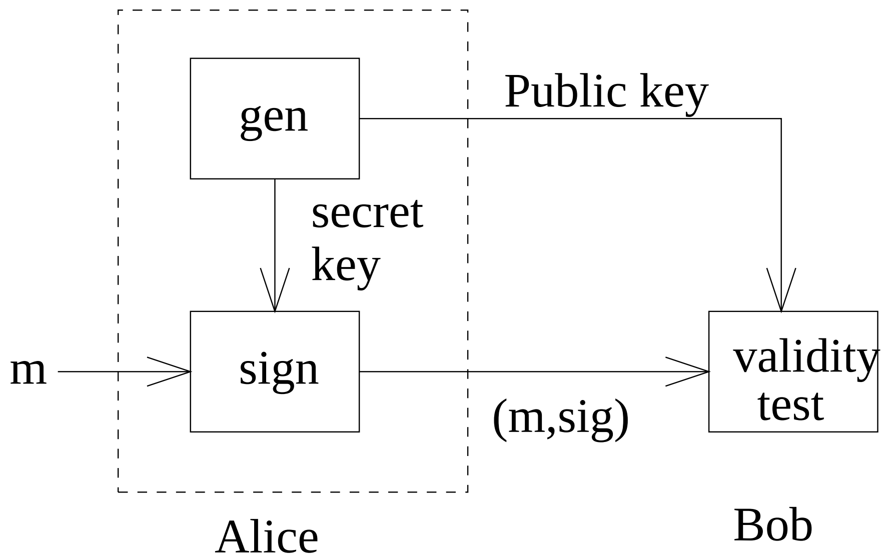
4. DIGITAL SIGNATURE

Scenario:



Transferable msg Authentication

Classical Protocol



⇒ Desirable Properties

(1) Not forgeable

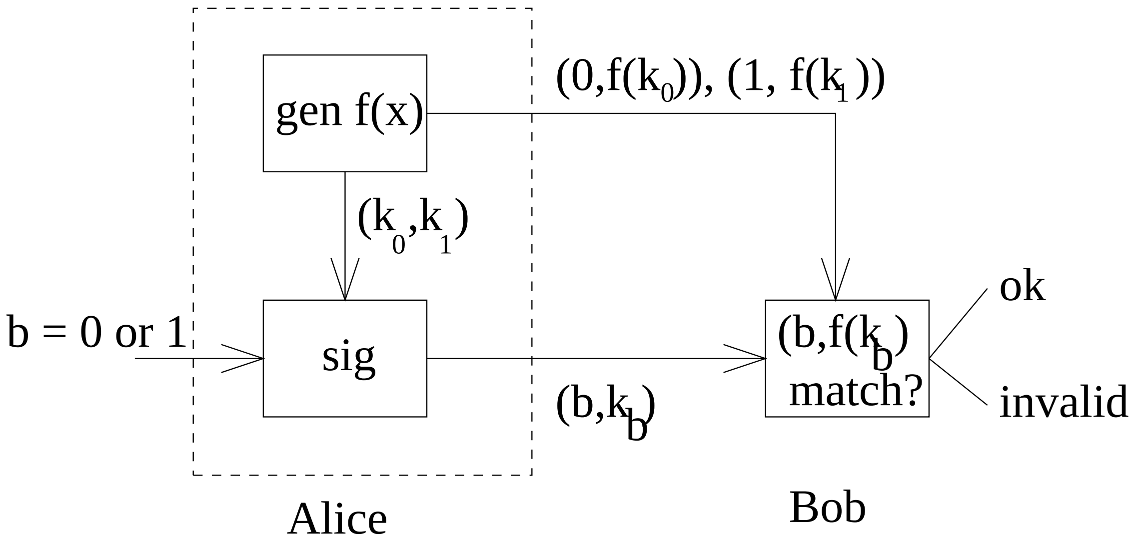
(2) Non-repudiatable

(3) Efficient w.r.t. signature size (Keys reusable)

4.1. One Time Classical Digital Signature Scheme (Lamport '79)

let f(x) be a one-way function (if you have x, getting $f(x)$ is easy. If you have $f(x)$, you can't find x. $f(x) \neq f(y)$, or is exponentially rare.

$f(x)$ is public knowledge



example:

$f([x,t]) = xy$ $k_0 = [7,13]$, $f(k_0) = 91$ $k_1 = [3,17]$, $f(k_1) = 51$

Public Key: (0,91),(1,51)

msg's that you can send are [0,[7,13]] or [1,[3,17]]. Once sent, you've burned your key. It is not efficient.

Rompel: 1990: Info-secure DSS $\Leftrightarrow$ One way function

Currently, we settle for computationally secure, but not informationally theoretic secure.

5. QUANTUM DIGITAL SECURITY SCHEME

Def:

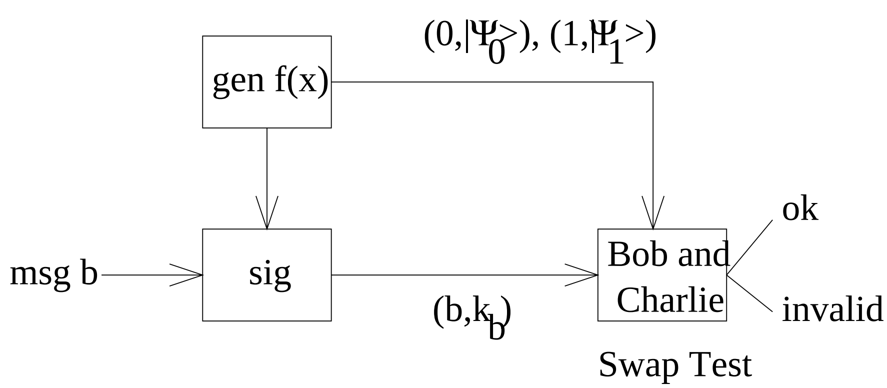
$k \mapsto |\Psi_k\rangle$

k has l bits. $|\Psi_k\rangle$ is the quantum fingerprint states from earlier, and has n qubits. $n \sim O(\log l)$

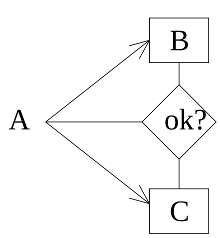
Claim:

This is a one way function, by Holevo's Thm. Can only extract n classical bits, given no prior entanglement.

Protocol:



Problem		Solution
Equality Test is probabilistic	$\Rightarrow$	Repeat: use m keys for each b
$|\Psi_k\rangle$ leaks $\log l$ bits about k	$\Rightarrow$	Limit copies of public key to $T < \frac{L}{N}$ (so, only T people can verify it)
Are all Public keys the same?	$\Rightarrow$	Symmetry test need certificate authority to check and distribute



$\Rightarrow$ Main Result

Info-theoretic one time public key secure D.S.S. whose classical msg b is signed by a classical private key string $(\tilde{k}, b)$ corresponding to a public quantum key $|\Psi_{\tilde{k}b}\rangle$

Resources Used

Size of b = 1 bit

$\tilde{k_b} = O(LM)$ bits

$|\tilde{\Psi}_k\rangle = O(m \log L)$ qubits

copies $|\Psi_{\tilde{k}}\rangle \leq \frac{L}{\log L}$

Security

Prob[successful forgery] $\leq e^{-(1-\frac{C_2}{1-\delta^2})M}$

Prob[successful repudiation] $\leq e^{-|C_2-C_1|\sqrt{M}}$

where C_2 and C_1 are constants.

Problems

How to reuse keys?

How to reduce to using no Q.C. or Q. Memory

What are C_1 and C_2? (Existence proved by Gottesman and Chuang)

Physical Implementation