Problem Set 8

Exercise 8.1
Demonstrate that Poisson's ratio, $\nu$, cannot exceed $\frac{1}{3}$ by finding particular value of strain $\varepsilon_{ij}$ (for an isotropic material with elastic Young's modulus $E > 0$) that makes the stored elastic strain energy negative.

Exercise 8.2
Starting with the Gibbs-Duhem expression for phases with fixed composition, derive the Clausius-Clapeyron relation $dP = (\Delta S / \Delta V)dT$.

Using a carefully worded sentence or two, describe what this Clausius-Clapeyron means physically.

Exercise 8.3
Consider a binary alloy with components $A$ and $B$, let $X_A^\alpha$, $X_A^\beta$, and $X_A^\gamma$ represent the compositions of three phases $\alpha$, $\beta$, and $\gamma$ that coexist at a triple point at $P = P_T$ and $T = T_T$.

Note that, for each phase in a binary alloy, the composition is given by one variable only because $X_A^\alpha = 1 - X_B^\alpha$, $X_A^\beta = 1 - X_B^\beta$, and $X_A^\gamma = 1 - X_B^\gamma$.

Starting with the Gibbs-Duhem expression, derive a relationship for the change in the triple point $dP_T = (\text{material properties})dT_T$.

Also for the triple point, find a relationship between the change in the chemical potential of $A$ ($d\mu_A$) and the change in the chemical potential of $B$ ($d\mu_B$).

Exercise 8.4
In Homework problem 5.2, you found the equilibrium temperature and length of a thermally expanding in contact with a thermostat.

Using the engineering solution in the solution set, determine whether that stability is locally stable or unstable.