Problem 1

In class some time ago we solved the fluid flow problem for Newtonian fluid flowing down a ramp with a slope of $\theta$ degrees and a thickness of $L$. Let’s revisit this problem for a non-Newtonian fluid with a flow rate exponent of ‘m’.

First, write down the steady-state flow equation and note how it is different when ‘m’ is added. Write down the boundary conditions, too. Integrate the equation once to obtain an expression for $dv/dx$.

Problem 2

Before integrating a second time, apply one of the boundary conditions to simplify the equation. Then, integrate again. Apply the second boundary condition and give the complete solution.

Problem 3

Make some plots of your solution to Problem 2, showing how the flow profile changes from Newtonian flow into the non-Newtonian regime. Include the Newtonian solution we found during the first fluid flow lecture.
Problems 4 & 5:

You are designing a factory to blow mold Al bottles. You have a choice of alloys:

<table>
<thead>
<tr>
<th>alloy</th>
<th>Melting point, C</th>
<th>Maximum strain rate, 1/s</th>
<th>m</th>
<th>composition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al 5083</td>
<td>500-550</td>
<td>5x10e-4 – 10e-3</td>
<td>0.65</td>
<td>4.4% Mg, 0.7% Mn, 0.12% Cr</td>
</tr>
<tr>
<td>Al 6061</td>
<td>500-610</td>
<td>10e-5 – 10e-2</td>
<td>0.9</td>
<td>1.0% Mg, 0.6% Si, 0.28% Mn, 0.2% Cr</td>
</tr>
<tr>
<td>Al 7091</td>
<td>300-500</td>
<td>8x10e-5 – 2x10e-2</td>
<td>0.38</td>
<td>6.5% Cu, 2.5% Mg, 1.5% Cu, 0.4% Co</td>
</tr>
<tr>
<td>Al 7475</td>
<td>515</td>
<td>2x10e-5 – 2x10e-2</td>
<td>0.9</td>
<td>5.7% Zn, 2.2% Mg, 1.6% Cu, 0.22% Cr</td>
</tr>
</tbody>
</table>

You begin with blanks (the things that look like test tubes) that are the width of a typical bottle cap and they must be blown to the size of a typical bottle with 100 um thickness. Your customers want grooves on the bottles, which introduce perturbations as the bottle is blown out. Some preliminary testing reveals that the perturbation size relative to the original thickness obeys the following function: 

\[(\text{perturbed region thickness/original thickness}) = \text{Exp}[-290 \text{ m}^{-1} \text{ (groove depth)}] \]

**Problem 4**

Compare the fracture strain for each alloy to an estimate of the total strain you expect a smooth-walled bottle to experience during blow-molding.

**Problem 5**

Determine the maximum groove depth tolerable for each alloy. Discuss your conclusions about making metal bottles from problems 4 and 5.