Radiation:

\[-k \frac{\partial T}{\partial x} = -\varepsilon \sigma \left[ T_{surf}^4 - T_{source}^4 \right] \]

\[ M = \varepsilon \sigma \frac{L}{k} T_{surf}^3 \]

\[ \frac{\partial T}{\partial t} = \alpha \nabla^2 T \]

\[ q = \varepsilon \sigma \left( T_{surf}^4 - T_{source}^4 \right) \]

⇒ Very few analytical solutions, some charts
CVD: Chemical Vapor Deposition

At steady state the thermocouple outputs a temperature reading $T_{TC}$

TC is heated by gas (convection)

\[ -h(T - T_f)A = \varepsilon \sigma (T_{surf}^4 - T_{source}^4)A \]

\[ T_{TC} = T_f - \frac{\varepsilon \sigma}{h} (T_{surf}^4 - T_{source}^4) \]

\[ T_{TC} \neq T_f \]

$T_f \approx 1000^\circ C$, $T_{wall} \approx 500^\circ C$,

$\varepsilon = 0.1$, $h = 100 W m^2 K$

\[ T_{TC} \approx 830^\circ C, \Delta T \approx 200^\circ C \]

Conclusions:

1. Objects that “see” cold surroundings are colder than you think
2. If an object “sees” a hot source it can be unexpectedly hot
3. “In vacuum” indicates no convection → radiation must be important

Topics Covered So Far:

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Next step is to discuss heat transfer combined with diffusion and phase transformations.

**Solidification:** Heat transfer plus phase transition, single component solidification

What are the B.C at the solid/liquid interface?

1. \( T = T_m \)
2. heat balance:
   \[
   -k_s \frac{\partial T}{\partial x}\Bigg|_s = -k_l \frac{\partial T}{\partial s}\Bigg|_l
   \]
3. heat of fusion

Look closely:

\( q_{in} = -k_l \frac{\partial T}{\partial x}\Bigg|_{x=s,l} \)
\[ q_{\text{out}} = -k_s \frac{\partial T}{\partial x} \bigg|_{x=s,s} \]

Fusion: \[ -H_f \left[ \frac{k_J}{kg} \right] \]

In time \( \Delta t \) the interface moved \( \Delta s \Rightarrow \) Volume transformed = \( A\Delta s \)

\[
\left( -k_i \frac{\partial T}{\partial x} \right) A + \left( k_s \frac{\partial T}{\partial x} \right) A - H_f \rho \left( \frac{\Delta s}{\Delta t} \right) A = 0
\]

where \( \frac{\Delta s}{\Delta t} = \frac{\partial s}{\partial t} \) = interface velocity

\[
= -k_s \frac{\partial T}{\partial x} \bigg|_s - k_l \frac{\partial T}{\partial x} \bigg|_l

= \frac{k}{H_f \rho} \left[ \frac{\partial T}{\partial x} \bigg|_s - \frac{\partial T}{\partial x} \bigg|_l \right] \text{ (within factor of two)}
\]

\[
\frac{\partial T}{\partial t} = \alpha_l \nabla^2 T \quad \frac{\partial T}{\partial t} = \alpha_s \nabla^2 T \quad \frac{\partial T}{\partial t} = \alpha_m \nabla^2 T
\]

\[
\frac{\partial s}{\partial t} = \frac{k}{H_f \rho} \left[ \frac{\partial T}{\partial x} \bigg|_s - \frac{\partial T}{\partial x} \bigg|_l \right]
\]

\[
T_l = T_s = T_m (m \propto H)
\]

\[
\frac{\partial s}{\partial t} = -k_s \frac{\partial T}{\partial x} \bigg|_s = -k_m \frac{\partial T}{\partial x} \bigg|_m
\]