Honeycomb-like materials in nature: wood

- "materials" derives from Latin "materies, materia" means wood or trunk of a tree
- old Irish - names of first letters of the alphabet refer to woods

\[\begin{align*}
A & \text{ aleu = elm} \\
B & \text{ beith = birch} \\
C & \text{ coll = hazel} \\
D & \text{ daiv = oak}
\end{align*}\]

Wood - Structure

- orthotropic (if neglect curvature of growth rings)
- \(\rho/\mu\) ranges from 0.05 (balsa) to 0.80 (lignum vitae)
- trees have cambial layer, beneath bark
- cell division @ cambial layer
  - new cells on outer part of cambial layer \(\rightarrow\) bark
  - """""" inner """""" \(\rightarrow\) wood
Wood structure

- living plant cells - plasma membrane + protoplast
- living cells secrete plant cell wall - analogous to extracellular matrix in animal tissues
- in trees, cells lay down cell wall over a few weeks, then die
- always retain a cambial layer of cells

**Cellular structure: softwoods**

- tracheids - bulk of cells (90%), provide structural support
  - have holes in cell wall for fluid transport (pits)
  - ~ 2.5 - 7.0 mm long; 20 - 80 μm across; t = 2 - 7 μm

- rays - radial arrays of smaller parenchyma cells that store sugars

**Cellular structure: hardwoods**

- fibers provide structural support; 35 - 70% of cells
- vessels - sap channels - conduction of fluids; 36 - 55% of cells
- rays - store sugars; 10 - 30% of cells
Softwood: Cedar

Hardwood: Oak

Structure: cell wall

- fiber reinforced composite
- cellulose fibers in matrix of lignin/hemicellulose
- 4 layers, each with fibers at different orientation
- between 2 cells: middle lamella

Cell wall properties

- Similar in different species of wood
  
  \[ \rho_s = 1500 \text{ kg/m}^3 \]
  
  \[ E_{SA} = 35 \text{ GPa} \]
  
  \[ E_{ST} = 10 \text{ GPa} \]
  
  \[ \sigma_{YSA} = 350 \text{ MPa} \]
  
  \[ \sigma_{YST} = 135 \text{ MPa} \]

(Note: cellulose: \( E \sim 140 \text{ GPa} \)

\( \sigma_y \sim 750 \text{ MPa} \))

\( A = \text{axial direction} \)

\( T = \text{transverse direction} \)
Wood Structure

(a) cellulose molecule

(b) cellulose macrofibril

(c) non-crystalline regions

(d) microfibril in amorphous hemicellulose and lignin

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Stress-strain curves

- σ-ε curves resemble those for honeycombs
- mechanisms of deformation best easily identified on low density balsa
curves + images for balsa
- tangential loading: formation of plastic hinges in bent cell walls
- radial loading: rays act as reinforcing; plastic yielding in cell walls
  - starts at pith, moves inward
- axial loading: axial deformity of cell walls, then break end caps
  - serrations correspond to each layer of end caps breaking

failure by plastic buckling + formation of kink bands also observed

- denser species

  Douglas fir - tangential, radial compression

  Norway spruce - axial compression
Stress strain curves

Balsa

Balsa: Tangential

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Balsa: Radial

Balsa: Axial

Douglas Fir: Tangential Comp

Douglas fir: Radial comp.

Norway spruce: Axial comp

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Data for wood

\[ \frac{E^*}{E_s} \propto \frac{\rho^*}{\rho_s} \quad \text{(axial)} \]

\[ \frac{E^*}{E_s} \propto \left( \frac{\rho^*}{\rho_s} \right)^3 \quad \text{(tangential; radial somewhat stiff)} \]

\[ \frac{\sigma^*}{\sigma_y^*} \propto \left( \frac{\rho^*}{\rho_s} \right) \quad \text{(axial)} \]

\[ \frac{\sigma^*}{\sigma_y^*} \propto \left( \frac{\rho^*}{\rho_s} \right)^2 \quad \text{(tangential/radial)} \]

\[ V_{rt}^* \approx 0.5 - 0.8 \quad V_{ra}^* \approx 0.02 - 0.07 \quad V_{ae}^* \approx 0.25 - 0.5 \]

\[ V_{re}^* \approx 0.2 - 0.6 \quad V_{ta}^* \approx 0.01 - 0.04 \quad V_{at}^* \approx 0.35 - 0.5 \]

Modelling wood properties

- Very simplified model - first order
- Does not attempt to capture finer details (e.g., softwoods vs. hardwoods)
- Cell wall has been modelled as fiber composite; it is itself anisotropic
- We normalize all properties with respect to \( E_s, \sigma_y^*, \) axial
- Constant of proportionality also reflects cell wall anisotropy
Figure courtesy of Lorna Gibson and Cambridge University Press.
Model for wood microstructure

Linear elastic moduli

- tangential loading - model as honeycomb - cell wall bending
  \[ E_T^* / E_s \propto (\rho^*/\rho_s)^3 \]
  - rays, end caps act to stiffen wood - data lie slightly above \((\rho/\rho_s)^3\)
- radial loading - rays act as reinforcing plates + are higher density than fibers
  \[ E_R^* = V_R R^3 E_T^* + (1 - V_R) E_T^* \]
  \[ V_R = \text{volume fraction of rays} \]
  \[ R = (\rho^*/\rho_s) \text{rays} / (\rho^*/\rho_s) \text{fibers} \approx 1.1 \text{ to } 2 \]
  \[ E_R^* \approx 1.5 E_T^* \]
  - \( E_R^* \) slightly larger than \( E_T^* \); \( \propto (\rho/\rho_s)^3 \)
- axial loading
  - axial deformation in cell wall
  \[ E_A^* / E_s \propto (\rho^*/\rho_s) \]

- explains, to first order, density dependence
  - anisotropy
Modelling - Poisson's ratios

\[ \nu^*_{R^T} = 0.5 - 0.8 \]
\[ \nu^*_{T^R} = 0.2 - 0.6 \]
\[ \nu^*_{R^A} = 0.02 - 0.07 \]
\[ \nu^*_{T^A} = 0.01 - 0.04 \]
\[ \nu^*_{A^R} = 0.25 - 0.5 \]
\[ \nu^*_{A^T} = 0.35 - 0.5 \]

Model

1  constraining effect of rays + end caps
0

data close to 0.4 ~ \nu

Modelling - compressive strength

- tangential loading - bending, plastic hinges
  \[ \sigma^*_T = \frac{16\gamma_5}{k} \left( \frac{\sigma^*}{\beta} \right)^2 \]

- radial loading
  \[ \sigma^*_R = V_R k^2 \sigma^*_T + (1 - V_R) \sigma^*_R \]
  
  balsa: \( V_R \approx 0.14 \) \( R \approx 2 \) \( \sigma^*_R = 1.4\sigma^*_T \)
  
  higher density woods - \( R \) smaller
  
  \( \sigma^*_R \) slightly larger than \( \sigma^*_T \); both \( \propto \left( \frac{\sigma^*}{\beta} \right)^2 \)

- axial loading - initial failure by axial yield (then end cap fracture, or buckling)
  \[ \sigma^*_A / \sigma_{ys} \propto \rho^*/\rho_s \]
Modelling: cell wall + cellular structure

- cell wall can be modelled as a fiber composite
  - cellulose $E \approx 140 \text{ GPa}$
  - lignin/hemicellulose $E \approx 2 \text{ GPa}$
- composite upper + lower bounds give envelope at right of figure
- measured values for $E_{\text{Axial}} = 35 \text{ GPa}$, $E_{\text{Transverse}} = 10 \text{ GPa}$
- can also show cellular solid model on same plot
- overall plot shows how wood hierarchical structure, density variation
  give wood moduli that vary by a factor of 1000
  
- can make similar plot for strength
Wood: Honeycomb Models

\[
\frac{E^*}{E_{s\text{along}}} = \frac{\rho^*}{\rho_s}
\]

\[
\frac{E^*}{E_{s\text{across}}} = \left(\frac{\rho^*}{\rho_s}\right)^3
\]

Wood: Honeycomb Models

Material selection

- for a beam of a given stiffness, $P/\delta$, length, $l$, square cross-section with edge length, $t$, what material minimizes the mass, $m$, of the beam?

$$m = \rho \cdot t^2 \cdot l$$

$$\delta = \frac{PL^3}{CEI} \quad \frac{P}{\delta} = \frac{CEt^4}{l^3} \quad t^2 = \left[ \left( \frac{P}{\delta} \right) \frac{l^3}{CE} \right]^{1/2}$$

$$m = \rho \left[ \left( \frac{P}{\delta} \right) \frac{l^3}{CE} \right]^{1/2} \cdot l$$

to minimize mass, choose material with min. $\rho/E^{1/2}$ or maximize $E^{1/2}/\rho$.

- Material selection chart: plot log $E$ vs log $\rho$
- Line of constant $E^{1/2}/\rho$ shown in red on plot
- Materials with largest values of $E^{1/2}/\rho$ at upper left of plot
- Woods have similar values of $E^{1/2}/\rho$ as engineering composites
- Note that tree trunky branches loaded primarily in bending.
- Also, note from models, $(E^*)^{1/2} = \frac{E_s^{1/2}}{\rho_s} \cdot \left( \frac{\rho^*}{\rho} \right)^{1/2}$

$= \text{Performance index for wood higher than that for the solid cell wall}$
- Similarly for strength in bending
Wood in Bending: $E^{1/2}/\rho$

\[
\frac{(E^*)^{1/2}}{\rho^*} = \frac{(E_s)^{1/2}}{\rho_s} \left( \frac{\rho_s}{\rho^*} \right)^{1/2}
\]

Stiffness performance index for wood in bending is similar to that for best engineering composites

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Wood in Bending: $\sigma_f^{2/3}/\rho$

$$\left(\frac{\sigma_f^*}{\rho^*}\right)^{2/3} = \left(\frac{\sigma_{ys}}{\rho_s}\right)^{2/3} \left(\frac{\rho_s}{\rho^*}\right)^{1/3}$$

Strength performance index for wood in bending is similar to that for best engng composites

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Wood Use in Design

Historical example: 17th century wooden ships

- colonial times, importance of navies to colonial powers
- used particular species for different parts of ship, based on their properties
- oak - used for much of the hull, ribs, knees, planking = dense wood; stiff + strong
  - "straight oak" - straight pieces, cut from trunk
  - "compass oak" - curved pieces from trunk + branch, so that grain runs along curved, cut pieces = max E, @
- used for knees, wing transom - curved pieces of ship hull

- Eastern white pine - British Royal Navy used for masts, imported from New England
  - England had run out of tall straight trees for masts
  - strategic resource - ship speed, size depended on size of mast + sail area
  - Eastern white pine known for straight, tall trunks; some over 100' tall
- lignum vitae - densest wood; acts as own lubricant
  - used in block & tackle
  - also used in clock gears ~1760
  - John Harrison's chronometer - story of Longitude, Davy Jones
Figure removed due to copyright restrictions. See *The international book of wood*. Bramwell, M, ed. Artists House, 1982. pp 186-87.
Modern example: glue laminated timber

- glue long pieces of wood, typically 1-2" thick, together
- select strips to avoid defects (e.g., knots)
- glue-lam has better mechanical properties than sawn lumber.
- also, can make curved members by using curved molds & clamps during bonding process
  - grain runs along the curve - exploits high stiffness & strength of wood along the grain
  - architecturally attractive