Thermal properties of foams

- Closed cell foams widely used for thermal insulation
- Only materials with lower conductivity are aerogels (tend to be brittle and weak) + vacuum insulation panels
- Low thermal conductivity of foams arises from:
  - Low volume fraction of solid
  - High volume fraction of gas with low $\lambda$
  - Small cell size suppresses convection + radiation (through repeated absorption & reflection)

- Applications: buildings, refrigerated vehicles, LNG tankers
- Foams also have good thermal shock resistance since coeff. of thermal expansion of foam equal to that of the solid + modulus much lower ($E = \alpha \Delta T$; $\sigma = E \alpha \Delta T = \sigma_f$)
  - $\Rightarrow$ used as heat shields
- Ceramic foams used as firebrick - ceramic has high $T_m$
  - Foam - low $\lambda$ - low heat loss
  - Low heat capacity - lowers energy to heat furnace to temperature
  - Good thermal shock resistance - resists spalling
Thermal conductivity, \( \lambda \)

- steady state conduction (\( T \) constant with time)

  Fourier's law:  \( q = -\lambda \frac{dT}{dx} \)

  \( q \) = heat flux  [J/(m^2s)]
  \( \lambda \) = thermal conductivity  [W/(mK)]
  \( \frac{dT}{dx} \) = temperature gradient

    \( = i \frac{dT}{dx} + j \frac{dT}{dy} + k \frac{dT}{dz} \)

- non-steady heat conduction (\( T \) varies with time, \( t \))

  \( \frac{dT}{dt} = \alpha \frac{\partial^2 T}{\partial x^2} \)

  \( \alpha \) = thermal diffusivity  = \( \frac{\lambda}{\rho C_p} \)  [m^2/s]

  \( \rho \) = density
  \( C_p \) = specific heat = heat req'd to raise temp. of unit mass by 1 K

  \( \rho C_p \) = volumetric heat capacity  [J/(m^3 K)]

- values for \( \lambda \), see Table 7.1
### Table 7.1 Thermal conductivities and diffusivities

<table>
<thead>
<tr>
<th>Material</th>
<th>Thermal conductivity $\lambda$ (W/m K)</th>
<th>Thermal diffusivity $a$ (m$^2$/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Copper (solid)</td>
<td>384$^a$</td>
<td>$8.8 \times 10^{-5}$ a</td>
</tr>
<tr>
<td>Aluminium (solid)</td>
<td>230$^a$</td>
<td>$8.9 \times 10^{-5}$ a</td>
</tr>
<tr>
<td>Alumina (solid)</td>
<td>25.6$^a$</td>
<td>$8.2 \times 10^{-6}$ a</td>
</tr>
<tr>
<td>Glass (solid)</td>
<td>1.1$^a$</td>
<td>$4.5 \times 10^{-7}$ a</td>
</tr>
<tr>
<td>Polyethylene (solid)</td>
<td>0.35$^a$</td>
<td>$1.7 \times 10^{-7}$ a</td>
</tr>
<tr>
<td>Polyurethane (solid)</td>
<td>0.25$^c$</td>
<td></td>
</tr>
<tr>
<td>Polystyrene (solid)</td>
<td>0.15$^a$</td>
<td>$1.0 \times 10^{-7}$ a</td>
</tr>
<tr>
<td>Air</td>
<td>0.025$^a$</td>
<td></td>
</tr>
<tr>
<td>Carbon dioxide</td>
<td>0.016$^a$</td>
<td></td>
</tr>
<tr>
<td>Trichlorofluoromethane (CCl$_3$F)</td>
<td>0.008$^a$</td>
<td></td>
</tr>
<tr>
<td>Oak ($\rho' / \rho_k = 0.40$)</td>
<td>0.150$^a$</td>
<td></td>
</tr>
<tr>
<td>White pine ($\rho' / \rho_k = 0.34$)</td>
<td>0.112$^a$</td>
<td></td>
</tr>
<tr>
<td>Balsa ($\rho' / \rho_k = 0.09$)</td>
<td>0.055$^a$</td>
<td></td>
</tr>
<tr>
<td>Cork ($\rho' / \rho_k = 0.14$)</td>
<td>0.045$^a$</td>
<td></td>
</tr>
<tr>
<td>Polystyrene foam ($\rho' / \rho_k = 0.025$)</td>
<td>0.040$^b$</td>
<td>$1.1 \times 10^{-6}$ b</td>
</tr>
<tr>
<td>Polyurethane foam ($\rho' / \rho_k = 0.02$)</td>
<td>0.025$^b$</td>
<td>$9.0 \times 10^{-7}$ b</td>
</tr>
<tr>
<td>Polystyrene foam ($\rho' / \rho_k = 0.029$–$0.057$)</td>
<td>0.029–0.035$^{cd}$</td>
<td></td>
</tr>
<tr>
<td>Polyisocyanurate foam, (CFC-11) (rho' = 32 kg/m$^3$)</td>
<td>0.020$^d$</td>
<td></td>
</tr>
<tr>
<td>Phenolic foam, (CFC-11, CFC-113) (rho' = 48 kg/m$^3$)</td>
<td>0.017$^d$</td>
<td></td>
</tr>
<tr>
<td>Glass foam ($\rho' / \rho_k = 0.05$)</td>
<td>0.050$^a$</td>
<td></td>
</tr>
<tr>
<td>Glass wool ($\rho' / \rho_k = 0.01$)</td>
<td>0.042$^d$</td>
<td></td>
</tr>
<tr>
<td>Mineral fibre ($\rho' / \rho_k = 4.8–32$ kg/m$^3$)</td>
<td>0.046$^{cd}$</td>
<td></td>
</tr>
</tbody>
</table>

All values for room temperature.

References


Data for thermal conductivity and thermal diffusivity
thermal diffusivity, \(a\)

- materials with a high value of \(a\) rapidly adjust their temp. to that of surroundings, because they conduct heat rapidly in comparison to their volumetric heat capacity; do not require much energy to reach thermal equilibrium

- e.g. Cu \(a = 112 \times 10^{-6} \text{ m}^2/\text{s}\)
- nylon \(a = 0.09 \times 10^{-6} \text{ m}^2/\text{s}\)
- wood \(a = 0.082 \times 10^{-6} \text{ m}^2/\text{s}\)

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**Thermal conductivity of a foam, \(\lambda^*\)**

\(\lambda^*\) - contributions from:
- conduction through solid, \(\lambda_s^*\)
- " " gas, \(\lambda_g^*\)
- convection within cells, \(\lambda_c^*\)
- radiation through cell walls + across voids, \(\lambda_r^*\)

\[
\lambda^* = \lambda_s^* + \lambda_g^* + \lambda_c^* + \lambda_r^*
\]

- conduction through solid: \(\lambda_s^* = \eta \lambda_s (\rho_s^{\star}/\rho_s)\) \(\eta = \text{efficiency factor} \sim 2/3\)
- conduction through gas: \(\lambda_g^* = \lambda_g (1 - \rho_s^{\star}/\rho_s)\)
For example, 2.5% dense closed-cell polystyrene foam $\lambda^* = 0.040 \text{ W/mK}$

$\lambda_s = 0.15 \text{ W/mK} \quad \lambda_g = 0.025 \text{ W/mK (air)}$

$\lambda^*_s + \lambda^*_g = \frac{2}{3} (0.15) (0.025) + (0.025) (0.975) = 0.043 + 0.024 = 0.027 \text{ W/mK}$

- Most of conductivity comes from conduction through gas
- Foams for insulation blown with low $\lambda_g$ gases
- Problem with aging - low $\lambda_g$ gases diffuse out of foam over time, air diff. $\lambda_s^*$

**Convection within the cell**

- Hot gas rises and falls due to density changes with temperature
- Density changes - buoyancy forces
- Also have viscous forces from drag of gas as it moves past cell wall

**Convection important if Rayleigh number $\gtrsim 1000**

$$Ra = \frac{\rho g \beta \Delta T_c l^3}{\mu a}$$

- $\rho$ = density of gas
- $g$ = grav. accn
- $\beta$ = volume expansion
- $\Delta T_c$ = temp. diff. across cell
- $l$ = cell size
- $\mu$ = dynamic viscosity of gas
- $\mu = \text{gas} = \frac{\mu}{\text{air}}$ (viscosity)
- $a$ = thermal diffusivity
**Convection**

For \( Re = 1000 \):
- \( \text{Air} \)
- \( p = P_{atm} \)
- \( T = \text{room temp} \)
- \( \beta = \frac{1}{300} \, (\text{oK}^{-1}) \)
- \( \Delta T_c = 1 \, \text{oK} \)
- \( \mu_{av} = 2 \times 10^{-5} \, \text{Pa} \cdot \text{s} \)
- \( \rho_{av} = 1.2 \, \text{kg/m}^3 \)
- \( a_{av} = 2.0 \times 10^{-5} \, \text{m}^2/\text{s} \)

\[ \Rightarrow l = 20 \, \text{mm} \]

- Convection important if cell size > 20 mm
- Most foams: cell size < 1 mm \( \Rightarrow \) convection negligible

**Radiation**

- Heat flux passing by radiation, \( q^\circ \), from a surface at temperature \( T_i \)
  to one at a lower temp. \( T_o \), with a vacuum between them, is:

\[ q^\circ = \beta_i \sigma (T_i^4 - T_o^4) \quad \text{Stefan's law} \]

\( \sigma = \text{Stefan's constant} = 5.67 \times 10^{-8} \, \text{W/m}^2\text{K}^4 \)

\( \beta_i = \text{constant} (<1) \) describing emissivity of the surface

(\( \beta_i \) describes how much of the radiant flux emitted per unit area of sample relative to a black body radiator at same temp. & conditions; black body absorbs all energy; black body emissivity = 1)
- If put foam between two surfaces, heat flux is reduced, since radiation is absorbed by the solid + reflected by cell walls.

- Attenuation: \( q_r = q_i^* \exp(-k^* t^*) \)  
  
  \( k^* = \) extinction coeff. for foam  
  
  \( t^* = \) thickness of foam

- For optically thin walls + struts (\( t < 10 \mu m \)) (transparent to radiation):  
  
  \( k^* = \left( \frac{\rho_i}{\rho_s} \right) k_s \)

- Heat flux by radiation then:  
  
  \( q_r = \lambda_i^* \frac{dT}{dx} \)
  
  \( q_r = \rho_i \sigma (T_i^4 - T_o^4) \exp[-(\rho_i^* \rho_s) k_s t^*] = \lambda_i^* \frac{dT}{dx} \)

- Obtain \( \lambda_i^* \) using some approximations:
approximations:

\[ \frac{dT}{dx} \approx \frac{T_1 - T_0}{t^*} = \frac{\Delta T}{t^*} \]

\[ T_1^* - T_0^* = 4 \Delta T \frac{T^3}{t^*} \quad T^* = \left( \frac{T_1 + T_0}{2} \right) \]

\[ q_v = \beta_1 \sigma 4 \Delta T \frac{T^3}{t^*} \exp \left[ -\left( \frac{\rho}{\rho_s} \right) k_s t^* \right] = \lambda_r^* \frac{\Delta T}{t^*} \]

\[ \lambda_r^* = 4 \beta_1 \sigma \frac{T^3}{t^*} \exp \left[ -\left( \frac{\rho}{\rho_s} \right) k_s t^* \right] \]

as \( \rho \to \rho_s \) ↓, \( \lambda_r^* \) ↑

---

**Thermal conductivity**

- relative contributions of \( \lambda_s^* \), \( \lambda_j^* \), \( \lambda_r^* \) shown in Fig 7.1
  - largest contribution \( \lambda_j^* \)

- \( \lambda^* \) plotted against relative density Fig 7.2
  - minimum @ between \( \rho_s \) of 0.03 & 0.07
  - at which point \( \lambda^* \) only slightly larger than \( \lambda_j^* \)
  - at low \( \rho_s \), \( \lambda^* \) increases - increasing transparency to radiation (also, walls may rupture)

- tradeoff: as \( \rho_s \) ↓, \( \lambda_s^* \) ↓ but \( \lambda_r^* \) ↑
Thermal Conductivity

Cond. Vs. Relative Density

Cond. vs. Cell Size

\( \lambda^* \) plotted against cell size Fig 7.3

- \( \lambda^* \) increases with cell size
- radiation reflected less often

Note: aerogels
  - pore size < 100 nm
  - mean free path of air at ambient pressure = 68 nm
    ➔ avg. distance molecules move before collision with another molecule
  - aerogels: pore size < mean free path of air - reduces conduction through gas.

Specific heat \( C_p \)

- specific heat = energy req'd to raise temp. of unit mass by unit temp

\[ C_p^* = C_{ps} \quad [J/\text{kg} \cdot \text{K}] \]

Thermal expansion coefficient

\( \alpha^* = \alpha_s \) (consider foam as framework)

(but if closed-cell foam cooled dramatically - gas can freeze, collapsing the cells; or if heated - gas expands, increasing the internal pressure + strain)
Thermal shock resistance

- If material subjected to sudden change in surface temp. induces thermal stresses at surface + cracking + spalling.
- Consider material at $T_1$ dropped in b water at $T_2$ ($T_1 > T_2$)
- Surface temp. drops to $T_2$, contracting surface layers
- Thermal strain $\varepsilon_T = \alpha \Delta T$
- If surface bonded to underlying block of material - constrained to original dimensions

$$\sigma = \frac{E \alpha \Delta T}{1 - \nu} \quad \text{in the surface}$$

- Cracking/spalling when $\sigma = \sigma_f$
- $\Delta T_c = \frac{\sigma_f (1 - \nu)}{E \alpha}$ = critical $\Delta T$ to just cause cracking
  
  for foam: (open cells)
  $$\Delta T_{c*} = 0.2 \sigma_{fS} \left( \rho S \alpha S \right)^{3/2} (1 - \nu) = 0.2 \sigma_{fS} \left( \frac{\rho S}{\rho_*} \right)^{1/2} \frac{\nu S}{E_s} \Delta T_{cS}$$
  $$\Delta T_{c*} \propto \text{as foam density } \downarrow \quad \Delta T_{c*} \uparrow \quad \text{firebrick - porous ceramic}$$
Case study: optimization of foam density for thermal insulation

- There is an optimal foam density for a given thermal insulation problem.
- Already saw $A^*$ has a minimum as a $f(\rho^*/\rho_s)$
- Typically, have a constraint on the foam thickness, $t^* = \text{constant}$
  $\lambda^* = \frac{2}{3} (\rho^*/\rho_s) \lambda_s + (1 - \rho^*/\rho_s) \lambda_f^* + 4 \beta_1 \sigma T^3 t^* \exp[-K_s (\rho^*/\rho_s) t^*]$
- What is optimum $\rho^*/\rho_s$ for a given $t^*$?
  \[
  \frac{d\lambda^*}{d(\rho^*/\rho_s)} = 0 \Rightarrow (\rho^*/\rho_s)_{\text{opt}} = \frac{1}{K_s t^*} \ln \left[ \frac{4 K_s \beta_1 \sigma T^3 t^*}{\frac{2}{3} \lambda_s - \lambda_f^*} \right]
  \]

- As given thickness $t^*$ increases, $(\rho^*/\rho_s)_{\text{opt}}$ decreases.
- As $T$ increases, $(\rho^*/\rho_s)_{\text{opt}}$ increases.

  e.g. coffee cup $t^* = 3\text{mm}$ $(\rho^*/\rho_s)_{\text{opt}} = 0.08$ (see PP slide Table 7.3 for data used in calculation)
  refrigerator $t^* = 50\text{mm}$ $(\rho^*/\rho_s)_{\text{opt}} = 0.02$
Case Study:
Optimization of Relative Density

Case Study:
Optimum Relative Density

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Extinction coefficient of solid polymer, $K_s$</td>
<td>$5.67 \times 10^4 \text{ m}^{-1}$</td>
</tr>
<tr>
<td>Emissivity factor, $\beta_1$</td>
<td>0.5</td>
</tr>
<tr>
<td>Conductivity of solid polymer, $\lambda_s$</td>
<td>0.22 W/m K</td>
</tr>
<tr>
<td>Conductivity of gas, $\lambda_g$</td>
<td>0.02 W/m K</td>
</tr>
<tr>
<td>Mean temperature, $T$</td>
<td>300°K</td>
</tr>
<tr>
<td>Stefan’s constant, $\sigma$</td>
<td>$5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4$</td>
</tr>
</tbody>
</table>

Case study: insulation for refrigerators

- insulation reduces energy cost, but has a cost itself
- total cost = cost of insulation + cost of energy lost by heat transfer through walls
- objective function: minimize total cost
- given: \( x = \) thickness of insulation
  \( \Delta T = \) temp. diff. across insulation
  \( t_e = \) design life of refrigerator
  \( C_m = \) cost of insulation/mass
  \( C_E = \) " " energy/joule
  \( C_T = \) total cost/area

\[
C_T = x \rho^* C_m + \lambda \frac{\Delta T}{x} t_e C_E
\]

(heat flux \( q = \lambda \frac{\Delta T}{x} \frac{W}{m^2}\))

Define \( M_1 = \frac{1}{\rho^* C_m} \quad M_2 = \frac{1}{\lambda} \)

\[
\frac{C_T}{x} = \frac{1}{M_1} \left[ \frac{\Delta T}{x^2} t_e C_E \right] \frac{1}{M_2}
\]
two terms are equal when:

\[ M_2 = \left[ \frac{\Delta T}{x^2} t_e C_e \right] M_1 \]

\( x \) coupling constant

- family of parallel straight lines of constant value \( \frac{\Delta T}{x^2} t_e C_e \)

- Fig 13.11  \( \Delta T = 20^\circ \quad x = 10\text{mm} \quad C_e = \# 0.01/\text{m}5 \)

- two lines for \( t_e = 10 \text{ years} \) & \( t_e = 1 \text{ month} \)

  (note error in book \( t_e = 10\text{yrs} \) line should be moved over)

also plotted a set of curved contours - plots of \( C_l/x \)

- as move up to right of plot, the value of \( C_l/x \) ↓

- for \( t_e = 10 \text{ years} \) = phenolic foam \( \rho^* = 0.035 \text{ Mg}/\text{m}^3 \)

  \( t_e = 1 \text{ month} \) = EPS \( \rho^* = 0.02 \text{ Mg}/\text{m}^3 \)

  PP \( \rho^* = 0.02 \text{ Mg}/\text{m}^3 \)
Case Study: Insulation for Refrigerators
