Short-fiber composites

- Easier fabrication, especially complex curvatures

- Critical fiber length

\[ \sigma_{f, b} \] - fiber failure stress

\[ \delta = \delta^* \] = debond length

\[ \tau_i = \text{interfacial shear strength} \]

\[ l_c = 2\delta^* = \frac{\sigma_{f, b}}{2\tau_i} \]
Shear Lag Theory (Cox)

\[ \sigma_f + \frac{d\sigma_f}{dy} \cdot dy \]

\[ (\frac{d\sigma_f}{dy} \cdot dy) \frac{\pi d^2}{4} = -\tau_i (\pi d) \cdot dy \rightarrow \frac{d\sigma_f}{dy} = -\frac{4}{d} \tau_i \]

Shear force per unit length at \( r \):

\[ \tau = \frac{d}{2r} \cdot \tau_i \]

Shear strain in matrix:

\[ \gamma = \frac{d\gamma}{dr} = \frac{d}{2r} \cdot \tau_i \]

\[ u = \int \frac{d\gamma}{dr} \cdot dv \rightarrow u_f - u_i = \int \frac{d}{2} \frac{\tau_i}{G} \ln \frac{2R}{d} \]

\[ \rightarrow (\frac{d}{2n}) \frac{d^2\sigma_f}{dy^2} - \sigma_f = -E_f \varepsilon_f, \quad n = \sqrt{\frac{2G\mu}{E_f \ln \frac{2R}{d}}} \]

\[ \sigma_f = E_f \varepsilon_f + C \sinh \frac{2\mu y}{d} + D \cosh \frac{2\mu y}{d} \]

b.c. \( \sigma_f = 0 \) @ \( y = \pm \frac{R}{2} \)

\[ \sigma_f = E_f \varepsilon_f \left[ 1 - \frac{\cosh (na \frac{2y}{d})}{\cosh (na)} \right], \quad a = \frac{q}{d} \]
Shear Lag Theory

\[
E_i = \eta v \frac{E_f}{E_m} + \nu \frac{E_m}{E_f}
\]

\[
\eta = 1 - \frac{\tanh(\eta a)}{\eta a}
\]

\[
\eta = \sqrt{\frac{2Gm}{E_f m (\frac{z^2}{d})}}
\]

\[
a = \frac{d}{l}
\]

eq 30% E-glass in N66

\[
E_i = 20 \text{ MPa}, \quad d = 12 \mu m, \quad l = 1 \text{ mm}
\]

\[
N = \frac{\pi d^2}{4} \rightarrow \frac{2E}{d} \frac{\pi}{4v^2} = 1.618
\]

\[
a = \frac{d}{j} = \frac{10^{-3}}{12 \times 10^{-6}} = 83.3
\]

\[
\eta = \sqrt{\frac{2 (1.015 \times 10^{-3})^2}{7689 \text{ ln } (1.618)}} = 0.2356
\]

\[
\eta = 1 - \frac{\tanh[(0.2356)(83.3)]}{(0.2356)(83.3)} = 0.949
\]

\[
E_i = (0.949)(-3)(7689) + (0.7)(2769)
\]

\[
= 23.5 \text{ GPa}
\]